I

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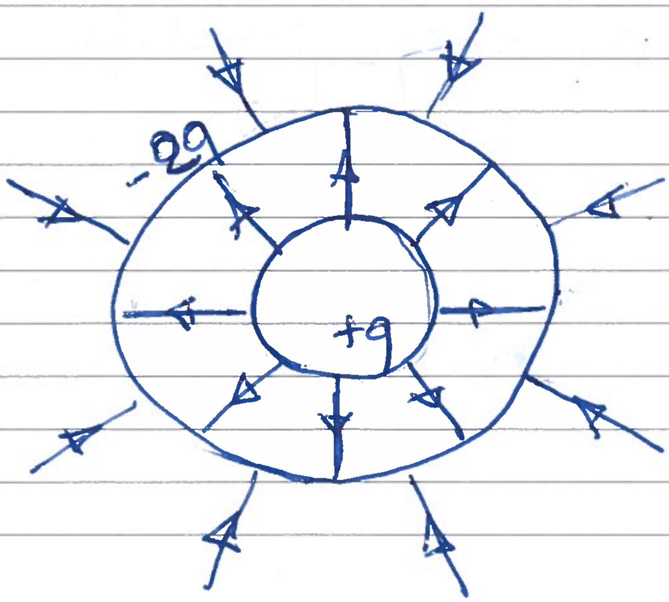
Problem I

1 a) attention points

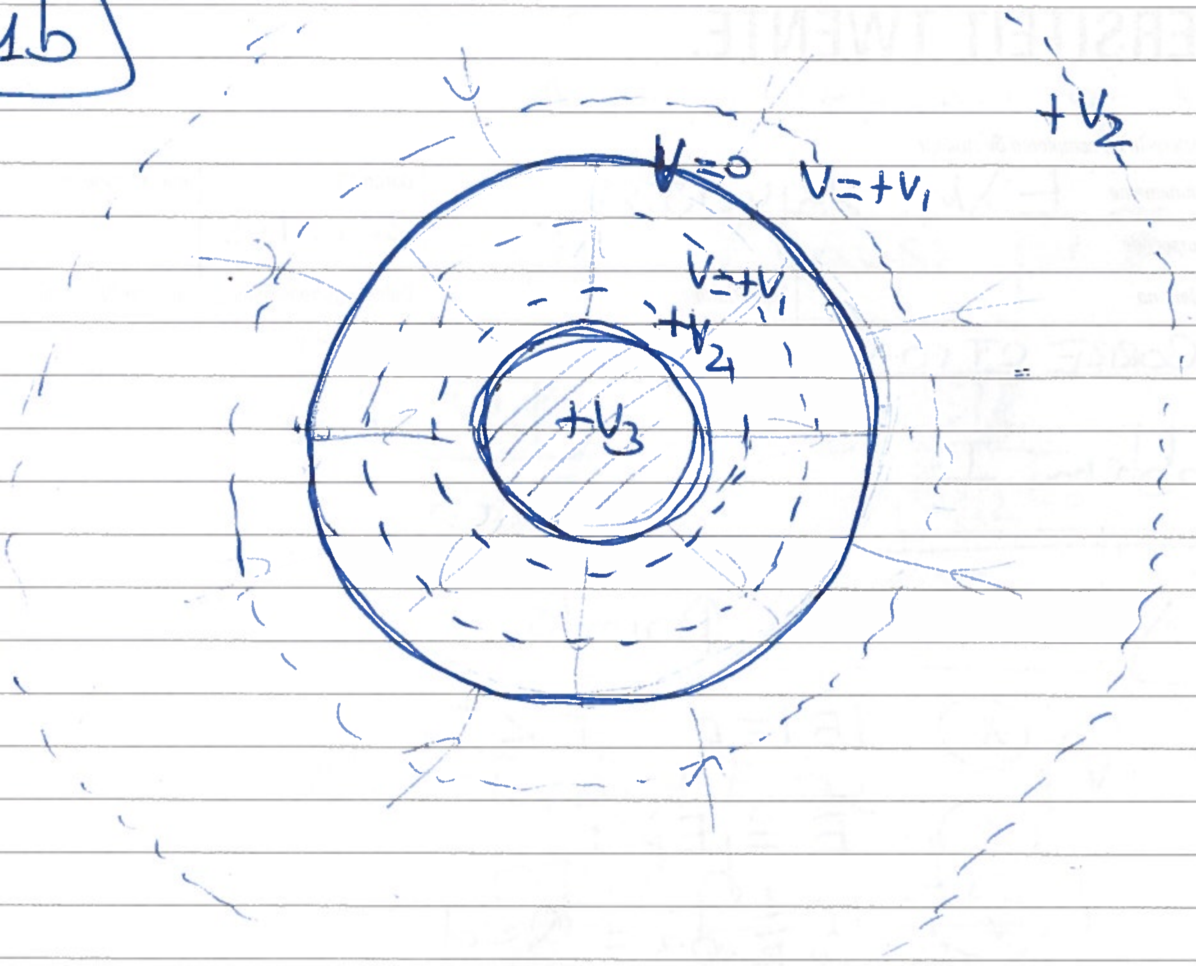
* $|\bar{E}| = 0 \quad r < R_1$
* $\bar{E} = E_r \, \hat{r}$
* $\oint \bar{E} \cdot d\bar{a} = \frac{Q_{encl}}{\epsilon_0}$

→ Same radial flux of field in-between shells ($Q_{encl} = +q$) and outside outer shell ($Q_{encl} = -q$), but opposite sign



II

1b)

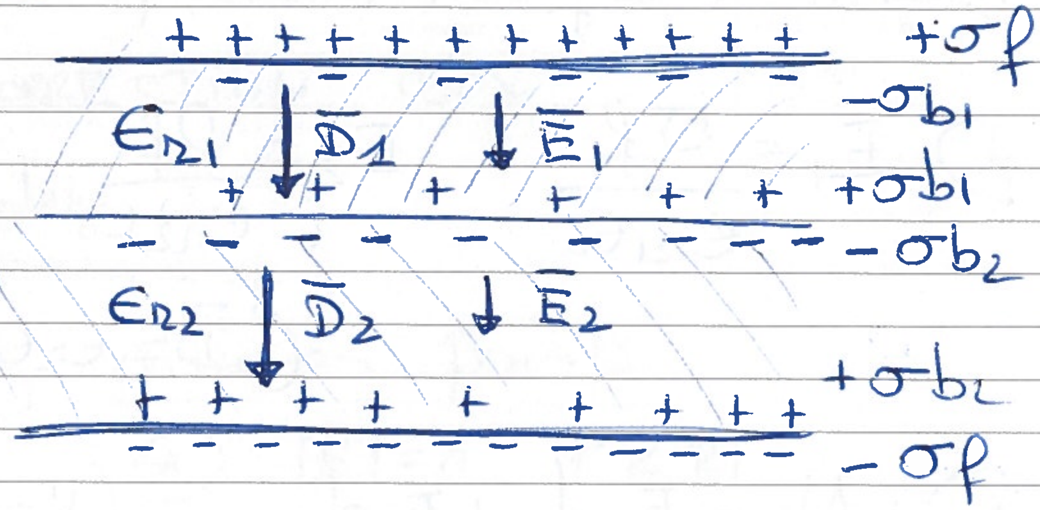


Attention points

- Concentric spherical surfaces
- + everywhere (except @ R_2)
- Distance increases radially
- $r < R_1$ = equipotential

Problem II

2a)



Attention points

* charge → location
  → signs
  → $\sigma_{b1} < \sigma_{b2} < \sigma_f$

* Fields → direction
  → $|\bar{D}_1| = |\bar{D}_2|$
  → $|\bar{E}_2| < |\bar{E}_1|$

2b) * $Q = C\Delta V$ (Definition of C)

* $|\bar{D}| = \sigma_f$ (Gauss for $\bar{D}$)

* $E_1 = \dfrac{\sigma_f}{\epsilon_{R1}\epsilon_0}$ $\quad E_2 = \dfrac{\sigma_f}{\epsilon_{R2}\epsilon_0}$

$(\bar{D} = \epsilon_R \epsilon_0 \bar{E})$

* $\Delta V = E_1 d_1 + E_2 d_2$ $\quad (V = -\int \bar{E} d\bar{e})$

$$= \frac{\sigma_f}{\epsilon_0}\left(\frac{d_1}{\epsilon_{R1}} + \frac{d_2}{\epsilon_{R2}}\right)$$

$$= \frac{\sigma_f}{\epsilon_0} \frac{d_1 \epsilon_{R2} + d_2 \epsilon_{R1}}{\epsilon_{R1}\epsilon_{R2}}$$

* $Q = \sigma_f A$

$$\rightarrow \boxed{C = \frac{Q}{\Delta V} = \frac{\cancel{\sigma_f} A \epsilon_0}{\cancel{\sigma_f}} \frac{\epsilon_{R1}\epsilon_{R2}}{\epsilon_{R1} d_2 + \epsilon_{R2} d_1}}$$

q.e.d.

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V

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Problem 3

3a : NT, inside the sphere the $\bar{E}$-field will depend on the details of $\rho(r)$ (e.g. charged shell)

3b : NT Gauss $\oint \bar{E} \cdot d\bar{a} = \frac{Q_{encl}}{\epsilon_0}$

If Q_{encl} is the same for cube & sphere, so will be Φ_E

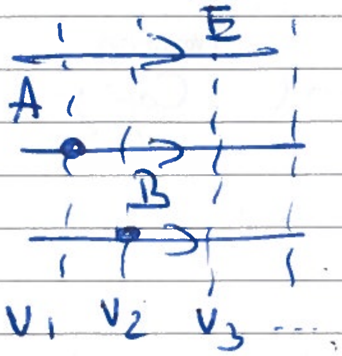
3c : T One has to exert a force $\bar{F}$ equal but opposite to the Coulomb force

$\bar{F}_{Coulomb} = Q_{test}\bar{E} = -Q_{test}\bar{\nabla}V$

$\rightarrow$ $\bar{F}$ along $+\bar{\nabla}V$

3d : NT

Counter-example : 2 points in a uniform $\bar{E}$-field



$V_A = V_1 \neq V_B = V_2$

$\bar{E}_A = \bar{E}_B$

3e : T

$\bar{\nabla} \times \bar{E} = 0 \rightarrow \bar{E} = -\nabla V$

$W_A = Q V_A$
$W_B = Q V_B$ & ΔV uniquely defined

$\rightarrow W_{AB} = W_B - W_A$ does not depend on the path followed

3f : T

Discoupled capacitor

$\rightarrow Q$ constant $\rightarrow \sigma$ constant

$\rightarrow |\bar{E}|$ constant

$\rightarrow |\Delta V| = |\bar{E}| d \nearrow$ when $d \nearrow$

(d = distance between plates)

3g : T the charges in the dipole partially "screen" each other → monopole $\sim 1/r^2$; dipole $\sim 1/r^3$; quadrupole $\sim 1/r^4$; ...

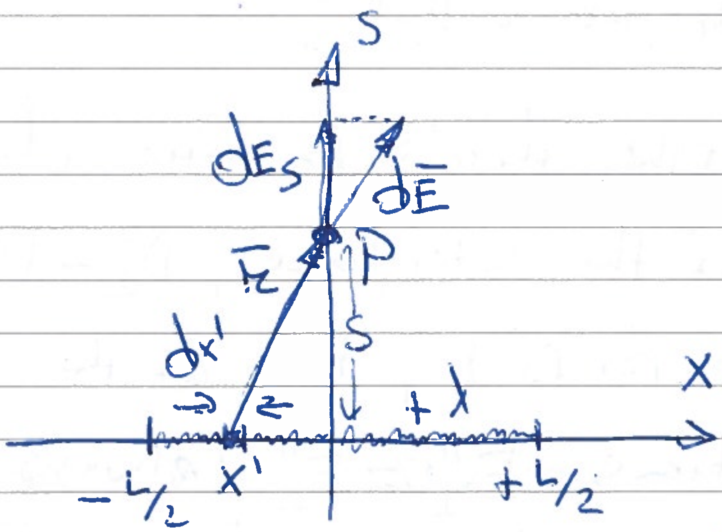
3h : NT when there's no free charge at the interface, $D_1^{\perp} = D_2^{\perp}$ (Gauss for $\bar{D}$), but at the same time $E_1^{\parallel} = E_2^{\parallel}$ (always true), so that in general ($\epsilon_{r1} \neq \epsilon_{r2}$) $D_1^{\parallel} = \epsilon_{r1}\epsilon_0 E_1^{\parallel} \neq D_2^{\parallel} = \epsilon_{r2}\epsilon_0 E_2^{\parallel}$

VIII

Problem 4

4a)

* Coulomb
$$d\bar{E} = \frac{dq}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}$$



$$dq = \lambda dx'$$
$$r^2 = x'^2 + s^2$$

$$\rightarrow d\bar{E} = \frac{1}{4\pi\epsilon_0}\frac{\lambda dx'}{x'^2+s^2}\hat{r}$$

* Symmetry $\rightarrow$ we need only dE_s, all other components cancel

$$\rightarrow \frac{dE_s}{|d\bar{E}|} = \frac{s}{|\bar{r}|}$$

$$\rightarrow dE_s = \frac{\lambda s}{4\pi\epsilon_0}\frac{dx'}{(x'^2+s^2)^{3/2}}$$

* $$\bar{E} = \left(\int_{x'=-L/2}^{+L/2} dE_s\right)\hat{s}$$

Standard Integral with $m=0$ & $n=-3/2$

$$\rightarrow \bar{E} = \frac{\lambda \cancel{s}}{4\pi\epsilon_0}\left[\frac{x'}{s^{\cancel{2}}\sqrt{x'^2+s^2}}\right]_{-L/2}^{L/2}\hat{s} = \boxed{\frac{1}{4\pi\epsilon_0}\frac{\lambda L}{s\sqrt{(L/2)^2+s^2}}\hat{s}}$$

q.e.d.

IX

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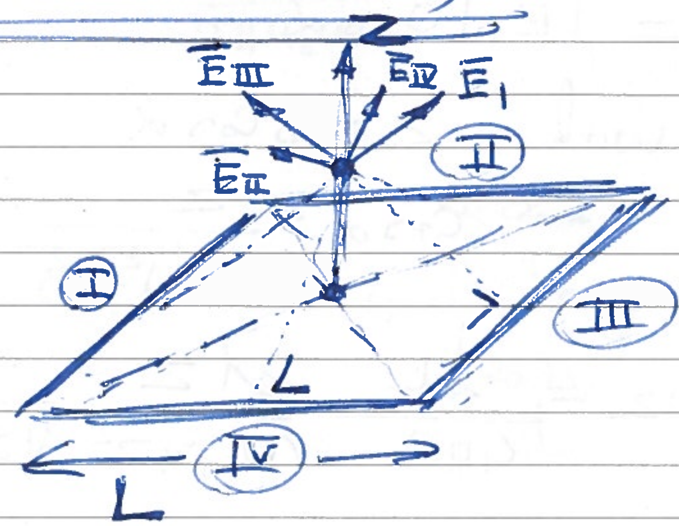
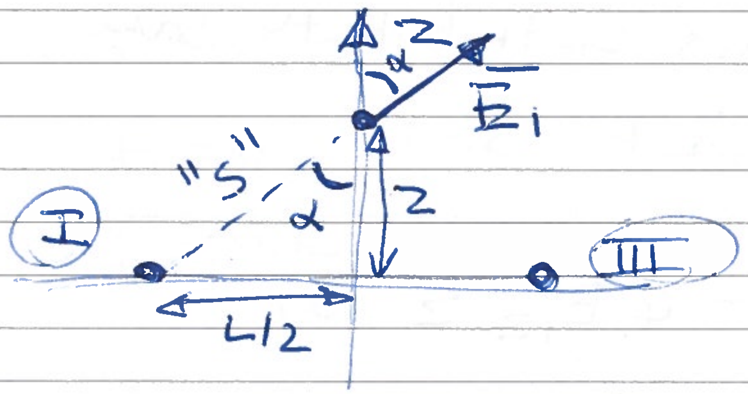
Problem 4 (continued)

4b)

Perspective



Side view (along I & III)



(*) Each side contributes an $\bar{E}$-field like in 4a, but we need to replace

$$\text{"}s\text{"} \longleftrightarrow \sqrt{(L/2)^2+z^2}$$

$$\rightarrow |\bar{E}_1| = \frac{\lambda L}{4\pi\epsilon_0}\frac{1}{\sqrt{(L/2)^2+z^2}}\frac{1}{\sqrt{2(L/2)^2+z^2}}$$

4b (continued)

* We only need the z-component, the other components cancel (symmetry)

$$E_{1z} = |E_1| \cos\alpha$$

with $z = s\cos\alpha$

$$\rightarrow \cos\alpha = \frac{z}{\sqrt{(L/2)^2+z^2}}$$

$$\rightarrow E_{1z} = \frac{\lambda L}{4\pi\epsilon_0} \frac{\cancel{1}z}{(L/2)^2+z^2} \frac{1}{\sqrt{2(L/2)^2+z^2}}$$

* All 4 sides contribute the same z-component

$$\boxed{\begin{aligned} \rightarrow \vec{E}_{tot} &= 4 E_{1z}\,\hat{z} \\ &= \frac{\lambda L}{\pi\epsilon_0} \frac{z}{(L/2)^2+z^2} \frac{1}{\sqrt{2(L/2)^2+z^2}}\,\hat{z} \end{aligned}}$$

Problem 5

XI

5a) ✱ $\rho_R = \rho_S = 0$ (Conductors)

✱ $\sigma_1 = \dfrac{+Q}{2\pi R_1 L}$ (all charge on surface of rod)

✱ $\sigma_2 = \dfrac{-Q}{2\pi R_2 L}$, so that for a virtual Gauss-cylinder inside the sleeve material $Q_{encl} = +Q - Q = 0$

✱ $\sigma_3 = 0$ (all charge $-Q$ is sitting already on the inside of the sleeve)

5b) Gauss $\rightarrow$

$\bar{E} = 0 \quad (s < R_1)$

$\bar{E} = \dfrac{Q/L}{2\pi\epsilon_0}\dfrac{1}{s}\hat{s} \quad (R_1 < s < R_2)$

$\bar{E} = 0 \quad (R_2 < s < R_3)$

$\bar{E} = 0 \quad (R_3 < s)$

5c)
$$V_b - V_a = -\int_a^b \bar{E} \cdot d\bar{\ell}$$

$$d\bar{\ell} = ds\,\hat{s}$$

$$\rightarrow V_{R_2} - V_{R_1} = -\int_{R_1}^{R_2} \frac{Q/L}{2\pi\epsilon_0} \frac{1}{s}\, ds$$

$$= -\frac{Q/L}{2\pi\epsilon_0} \Big[\ln s \Big]_{R_1}^{R_2}$$

$$= \frac{Q/L}{2\pi\epsilon_0} \ln\left(\frac{R_1}{R_2}\right)$$

$$\boxed{|\Delta V| = \frac{Q/L}{2\pi\epsilon_0} \ln\left(\frac{R_2}{R_1}\right)}$$

5d)
$$Q = C_0 \Delta V$$

$$C_0 = \frac{Q}{\Delta V}$$

$$\boxed{C_0 = \frac{L\, 2\pi\epsilon_0}{\ln\left(R_2/R_1\right)}}$$

XIII

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Problem 5 (continued)

5e) $\overline{D}_{new} = \overline{D}_{old} = \dfrac{Q/L}{2\pi s}\hat{s}$

$\rightarrow \boxed{\overline{E}_{new} = \dfrac{\overline{D}_{new}}{\epsilon_0 \epsilon_R} = \dfrac{Q/L}{2\pi\epsilon_0\epsilon_R}\dfrac{1}{s}\hat{s}}$

5f) $\overline{E}_{new}$ is a factor $\dfrac{1}{\epsilon_R}$ lower than $\overline{E}_{old}$

$\rightarrow$ ΔV_{new} is a factor $\dfrac{1}{\epsilon_R}$ lower

$\rightarrow$ C " " " ϵ_R higher

$\rightarrow \boxed{C/C_0 = \epsilon_R}$

Problem 5 (continued)

5g) $\bar{D} = \epsilon_0 \bar{E} + \bar{P}$

$$\rightarrow \bar{P} = \bar{D} - \epsilon_0 \bar{E}$$

$$= \frac{Q/L}{2\pi S}\left(1 - \frac{1}{\epsilon_r}\right)\hat{S}$$

$$= \frac{Q/L}{2\pi S}\,\frac{\epsilon_r - 1}{\epsilon_r}\,\hat{S}$$

$\sigma_b = \bar{P} \cdot \hat{n}$

$\rightarrow$ at R_1, $\hat{n} = -\hat{S}$ (outward from dielectric)

$$\boxed{\sigma_{b1} = \frac{-Q/L}{2\pi R_1}\,\frac{\epsilon_r - 1}{\epsilon_r}}$$

$\rightarrow$ at $S = R_2$, $\hat{n} = \hat{S}$

$$\boxed{\sigma_{b2} = \frac{+Q/L}{2\pi R_2}\,\frac{\epsilon_r - 1}{\epsilon_r}}$$

$\rho_b = -\bar{\nabla} \cdot \bar{P} = -\frac{1}{S}\frac{\partial}{\partial S}(S P_S)$ $(P_\phi = P_z = 0)$

$$= -\frac{1}{S}\frac{\partial}{\partial S}\left(\frac{Q/L}{2\pi}\,\frac{\epsilon_r - 1}{\epsilon_r}\right)$$

$$\rightarrow \boxed{\rho_b = 0}$$