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E&M ReTest 1
April 13th 2021

201300164 & 201300183
Fields and electromagnetism

Instructions

You have **2 hours** to complete the test. Clearly indicate your name and student number on every sheet that you hand in.

You may use a hand-written formula sheet containing maximum 10 equations. This sheet must be handed in together with your answers.

Before answering the questions, read all of them and start with the one you find easiest.

The amount of points to be obtained with each question is indicated next to the question number.

Problem 1 (20pts/100)

An charged metal sphere (with radius R_1 and charge +Q) is surrounded by two concentric thick metal shells (one with inner and outer radii R_2 and R_3 ; the second one with R_4 and R_5). The net charge on the inner shell is zero, the outer metal shell carries a net charge -Q.

1.a Create a graph of the electric field strength as a function of the radius *r*, indicating R_1, R_2, .. R_5 on the axis.

1.b Sketch a cross-sectional view of the situation, indicating the direction of the E-field lines and the position of any net charges.

Briefly, an electrical connection is made between the two *shells* (i.e. between R_3 and R_4) allowing charge to re-distribute. As soon as a new equilibrium is established, the connection is removed.

1.c Repeat question 1.a, this time for the new situation.

1.d Work out the energy difference between the first and second situation.

Problem 2 (20pts/100)

Consider a flat disk of radius R_0 with a homogenous surface charge density σ.

2.a. Show that the field $\perp$ above its center is given by:

$$\mathbf{E} = E_z\hat{\mathbf{z}} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R_0^2}}\right)\hat{\mathbf{z}},$$

with z the distance to its center.

2.b. Describe the asymptotic form of this equation in the limits $z \ll R_0$ and $R_0 \ll z$, i.e. close-by or far away from the plate. What type of field do these asymptotic forms represent and why?

Hint: $(1+u)^n \approx 1+nu$ for $u \to 0$ and $\sigma\pi R_0^2 = Q_{tot}$

Problem 3 (20pts/100)

Below you find 8 statements. For each of them, indicate whether the statement is 'true' **(T)** or 'not true' **(NT)**. Also include a brief argument why you agree or not (**minimum** 1 & **maximum** 5 lines per statement). Read the statements carefully, each word may be important!

3.a. Consider three identical point charges that are placed on the corners of an equilateral triangle. When there's no other charge nearby, the electric field at the center of the triangle is zero.

3.b. When we double the charge of all point charges in a configuration, the electric field due to this configuration becomes four times as strong.

3.c. Any closed path-integral of a static electric field must equal zero.

3.d. Far away from an electrical dipole, the electric field strength on its axis decreases as $1/r^3$ (with r the distance to the dipole).

3.e. The total electric flux through a surface that encloses a charge distribution is determined by the shape of the surface AND by the total amount of enclosed charge.

3.f. The electric potential due to a uniformly and positively charged infinite flat plane decreases linearly with the perpendicular distance to that plane.

3.g. Inside an empty cavity within a conductor the electric field is always zero, regardless of the shape of the cavity and of the charge distribution outside the conductor.

3.h. The electric potential difference between the electrodes of a disconnected capacitor remains conserved when we insert a dielectric in-between the electrodes.

Problem 4 (25pts/100)

A long cylindrical *metal* rod with radius R carries a linear charge density $+\lambda > 0$ and is coated with a dielectric sleeve of thickness d and relative permittivity ε_r (Figure 1).

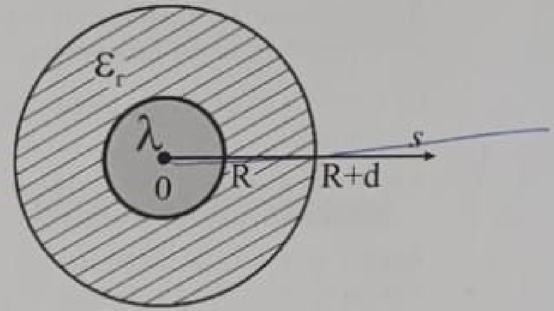

Figure 1: A charge-carrying metal rod in a dielectric sleeve (problem 4).

4.a Work out an expression for the electric field **E** as a function of s, the distance to the center of the rod.

Create a graph of the magnitude of **E** as a function of s. The sketch should at least cover the region $0 \le s \le R + 3d$ and on the vertical axis you should indicate some key values.

4.b Work out the electric potential V_0 on the central axis as function of the parameters λ, R, d and ε_r (take the potential on the outside of the sleeve as zero reference: $V(s = R + d) = 0$).

4.c Work out the free surface charge density σ_f (s = R) at the surface of the rod; the bound surface charge density σ_b (s = R) at the inner surface of the dielectric sleeve; the bound volume charge density $\rho_b(s)$ inside the sleeve; and the bound surface charge density σ_b (s = R+d) at the outer surface of the sleeve. (Once more as a function of the parameters λ, R, d and ε_r. Also give the sign, + or -.)

Problem 5 (15pt/100)

Electric field lines are refracted at the boundary between different dielectrics. Referring to figure 2, show that in the absence of free charge on the boundary it can be written that

$$\varepsilon_2/\varepsilon_1 = \tan\theta_2/\tan\theta_1.$$

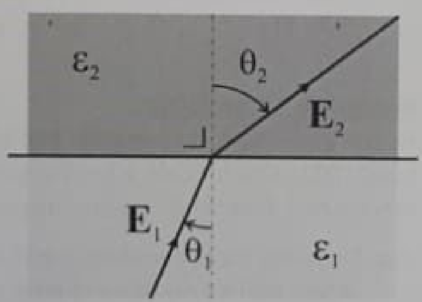

Figure 2: Refraction of an E-field at the interface between 2 dielectrics (problem 5).