E&M
UNIVERSITY OF TWENTE.

E&M ReTest 2
April 13th 2021

Boller, Offerhaus, Dhallé

201300164 & 201300183
Fields and electromagnetism

Instructions

You have **2 hours** to complete the test. Clearly indicate your name and student number on every sheet that you hand in.

You may use a hand-written formula sheet containing maximum 10 equations. <u>This sheet must be handed in together with your answers.</u>

Before answering the questions, read all of them and start with the one you find easiest.

The amount of points to be obtained with each question is indicated next to the question number.

<u>Problem 1 (20pts/100)</u>

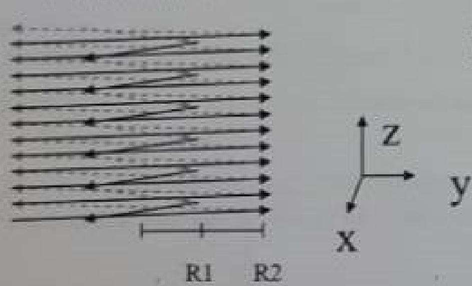
Consider two concentric coils oriented along the z-axis. The coils can be considered infinitely long. The outer coil has *2n* windings per meter, the inner one has only *n*. The coils are connected in such a way that the same current that runs up through the outer coil, runs back down through the inner coil. Both coils are wound in the same positive direction, viewed from the positive z-axis (both have a positive 'speed'). The wires may be considered infinitely thin.

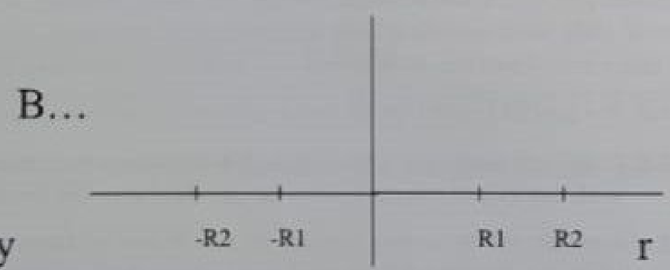
1.a Sketch the magnetic induction lines **B** in the cross-sectional view (copy the view onto your answer sheet).

1.b In the graphs indicate which component of the **B**- and **A**-field is non-zero and sketch the strength of that component in the graphs.

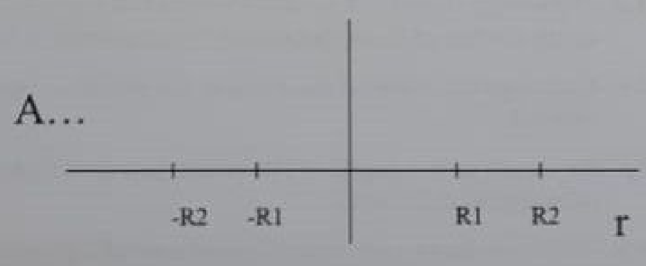
1.c Now an iron mantle ($\mu_r = 50$) is inserted such that it completely fills the space *between* the two coils. In the cross-sectional view, sketch the surface and/or volume *bound* currents. Make sure to indicate the direction of these currents.

Front view





Crosssection



Problem 2 (15pts/100)

A long and straight co-axial cable of length ℓ (figure 1) consist of a central conductor (with radius a) that carries a homogenous volume current density **J** and a mantle (radius b) that carries the same total current I back, distributed homogeneously as a surface current **K**.

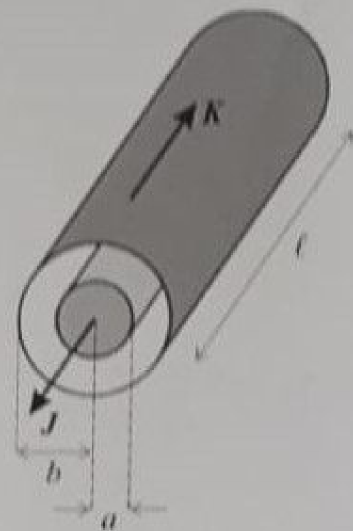


Figure 1: current-carrying coaxial cable

2a. Express **J** and **K** in the total current I and the radii a and b.

2b. Calculate the magnetic induction **B** inside the central conductor ($s < a$), in-between the conductors ($a < s < b$) and outside the conductors ($b < s$). Sketch the magnitude of **B** as a function of the distance s tot the central axis.

2c. Using the fact that $u = \dfrac{B^2}{2\mu_0}$ [J/m³] is the energy *density* stored in the field and that the total energy stored in a current-carrying self-inductor is $U = \dfrac{LI^2}{2}$ [J], show that the self-inductance L per unit length of the wire can be expressed as:

$$\frac{L}{\ell} = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \ln\left(\frac{b}{a}\right)\right] \text{ [H/m]}.$$

Problem 3 (20pts/100)

Below you find 8 statements. For each of them, indicate whether the statement is 'true' **(T)** or 'not true' **(NT)**. Also include a brief argument why you agree or not (**minimum** 1 & **maximum** 5 lines per statement). Read the statements carefully, each word may be important!

3.a. In a part of space that carries a uniform current density **J**, the magnetic flux density **B** is constant.

3.b. A charged particle that is injected in a uniform magnetic field will follow a 'corkscrew' trajectory with the axis of the corkscrew perpendicular to the **B**-field.

3.c. A metal disk is turning around its axis in een uniform magnetic field that points along this axis. Looking along the **B**-field lines, we see the disk turn clockwise. The center of the disk will become negatively charged.

3.d. The energy stored in a current-carrying coil depends on the current, on the shape of the coil and on its number of turns, but not on the properties of the material inside the coil.

3.e. A diamagnetic material placed near the end of a current-carrying coil will be pushed away from that coil.

3.f. The closed path integral of the vector potential A equals the magnetic flux through any surface delimited by that path.

3.g. When we double the cross-sectional area of a given current-carrying material while keeping the current density **J** through this section the same, the electric field will decrease by a factor 2.

3.h. In order to extract a copper plate from a perpendicular magnetic field, one needs to excert a pulling force.

Problem 4 (25pt/100)

4.a Calculate the magnetic induction **B** in a point P at a perpendicular distance z under the endpoint of a straight wire-segment of length ℓ that carries a current I.

4.b Make a sketch of the situation in which you indicate the direction of the current as well as the geometry and the direction of the **B**-field.

Problem 5 (20pts/100)

Consider two large and flat parallel plates that are spaced a distance d apart and that carry a uniform surface current density K_0 in opposite directions.

Take the mid-plane between the plates to be the xz-plane and assume the current in the top plate (at y = +d/2) to be $\mathbf{K} = -K_0\hat{\mathbf{z}}$, while in the bottom plate (at y = -d/2) $\mathbf{K} = +K_0\hat{\mathbf{z}}$ (figure 2). The space in-between the plates is half-filled (for $-d/2 < y < 0$) with a material with magnetic permeability μ, the rest is empty (permeability μ_0 for $0 < y < +d/2$ and for $|y| > d/2$). The plates may be considered as infinite in the x- and z-directions.

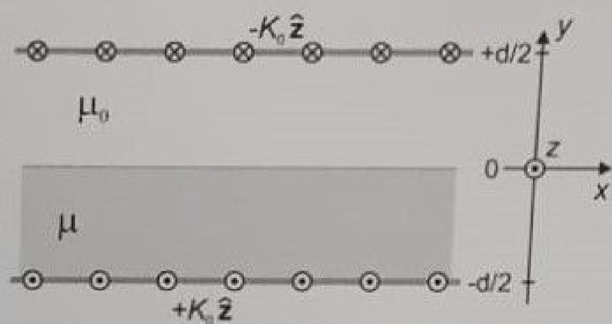


Figure 2: two current-carrying paralel plates

5.a Derive a vector expression for the fields **H**, **M** and **B** in the whole of space.

5.b Derive vector expressions for all bound magnetization currents $\mathbf{K}_b$ and $\mathbf{J}_b$. Make sure to clearly indicate their location and direction.

5.c Work out the magnitude and the direction of the magnetic pressure (i.e. the force per unit area) that acts on the top plate.