Boller, Offerhaus, Dhallé

E&M Test 2
April 19th 2022

201300164 & 201300183
Fields and electromagnetism

Instructions You have **2 hours** to complete the test. Clearly indicate your name and student number on every sheet that you hand in. You may use a hand-written formula sheet containing maximum 10 equations. This sheet must be handed in together with your answers.

Before answering the questions, read all of them and start with the one you find easiest.
The amount of points to be obtained with each question is indicated next to the question number.

Problem 1 (15 points/100)

Sketch the magnetic **B**-field lines for 3 different situations:

1.a. For a current-carrying ring of radius *R*. Sketch the field near the ring (for a distance *s* to the axis up to ~*3R*) and far away (*s* >> *R*).

1.b. For a short coil (with length *L* of the same order as the radius *R*). Sketch this for *s* up to ~*3R*.

1.c. For a flat ribbon (width *w* in the *x*-direction, thickness *d* ~ *w/10* in the *y*-direction, length *L* in the *z*-direction >> *w*). Sketch a cross-sectional *xy*-view for *x* and *y* up to ~*2L*.

Problem 2 (15pts/100)

Picture two cylindrical and concentric coils of infinite length. Looking along the central *z*-axis the current in the outer coil flows azimuthally in the clock-wise direction, while the same current flows back through the inner coil in the anti-clockwise direction. The radius of the outer coil is 2R, that of the inner coil R. The number of turns per unit length for the outer coil is 2n, i.e. twice as big as the number of turns per unit length for the inner coil (n). Calculate the magnetic induction **B** as a function of the radial distance *s* in all areas (inside the inner coil, in-between the coils and outside the outer coil). Make sure to include the direction of the field. Also make a sketch of the magnitude of the relevant field component as a function of *s*.

Problem 3 (20pts/100) Below you find 8 statements. For each of them, indicate whether the statement is 'true' **(T)** or 'not true' **(NT)**. Also include a brief argument why you agree or not (**minimum** 1 & **maximum** 5 lines per statement). Read the statements carefully, each word may be important!

3.a. A charge q that moves parallel to a magnetic field-line **B** will experience no Lorentz force.

3.b. A circular metal disk is turning around its perpendicular central axis within a uniform magnetic field that points along this same axis. Looking along the direction of the magnetic field, the disk is turning clock-wise. Due to this rotation, the center of the disk will become negatively charged with respect to its outer edge.

3.c. The observation of a magnetic **B**-field with a non-zero divergence would imply the existence of magnetic monopoles.

3.d. Two cylinders with identical radius and length are stacked axially on top of each other inside a long solenoid with a slightly larger radius. One cylinder is diamagnetic, the other one paramagnetic. When a current through the coil is switched on, these cylinders will move away from each other.

3.e. An extended ferromagnetic thick but flat plate has a remanent magnetization $\mathbf{M}_R$ perpendicular to the plane of the plate. The associate bound surface currents $\mathbf{K}_B$ on the two extended flat surfaces on either side of the plate flow in opposite directions.

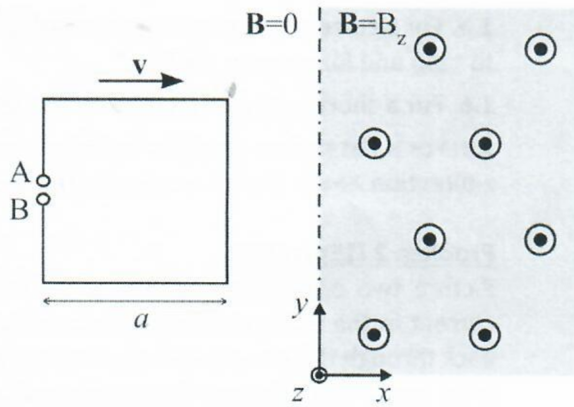
3.f. At the interface between two magnetic materials, the perpendicular component of the **H**-field must be continuous.

3.g. A flat copper plate is moving in a direction along its plane, from a region of space without magnetic field into a region with a magnetic **B**-field that points perpendicular to the plate. This will cause the plate to experience a force that points along its plane, but perpendicular to its velocity.

3.h. An electric current flows from a metal sphere with radius R to a concentric metal shell with inner radius 2R, with the space in-between filled with a medium with finite conductivity σ. The electric field in the medium just outside the surface of the sphere is twice as strong as the field in the medium just below its interface with the shell.

Problem 4 (25pts/100)

A square wire loop has sides of length a and moves frictionless with an initial speed $\mathbf{v} = v\hat{\mathbf{e}}_x$ from an area that has no magnetic field (x<0) into an area with a homogeneous magnetic induction $\mathbf{B} = B\hat{\mathbf{e}}_z$ (x>0). The loop has a finite mass m and a finite conductivity σ.



4.a Initially, the loop is open between A and B. Sketch the induced voltage $V_{AB} = V_A - V_B$ as the wireframe enters the field-area as a function of time. Pay attention to the sign of the voltage and mark the relevant points in time on the axis. The distance between A and B can be neglected compared to a.

4.b Repeat 4.a), but now for double the speed $\mathbf{v} = 2v\hat{\mathbf{e}}_x$. Use the same scale for time and voltage as in your previous sketch in a) and again mark all the relevant points.

4.c Again repeat 4.a), but now for a loop that has sides twice as big ($2a$) (and the original speed $\mathbf{v} = v\hat{\mathbf{e}}_x$). Again use the same scaling for the graph so that they can be compared.

4.d Now the loop is closed between A and B and again floats frictionless in the direction of the y-axis. Will the speed decrease linearly or exponentially as it enters the field region?

Problem 5 (20pts/100)

A thin hollow spherical shell of radius R carries a uniform surface charge density $+\sigma_e$ and spins around an axis through its center (the z-axis) with an angular velocity of ω.

5.a Find an expression for the surface current density K as a function of the angle Θ with the z-axis

5.b Calculate the **B**-field at the center of the shell.

VECTOR DERIVATIVES

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}; \quad d\tau = dx\,dy\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}; \quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient : $$\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

Curl : $$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian : $$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}; \quad d\tau = s\,ds\,d\phi\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial\phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial\phi}\right]\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}$$

VECTOR IDENTITIES

Triple Products

(1) $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})$

(2) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$

(5) $\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$

(6) $\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$

(7) $\nabla\times(f\mathbf{A}) = f(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla f)$

(8) $\nabla\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} + \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A})$

Second Derivatives

(9) $\nabla\cdot(\nabla\times\mathbf{A}) = 0$

(10) $\nabla\times(\nabla f) = 0$

(11) $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem : $\int_{\mathbf{a}}^{\mathbf{b}}(\nabla f)\cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem : $\int(\nabla\cdot\mathbf{A})\,d\tau = \oint\mathbf{A}\cdot d\mathbf{a}$

Curl Theorem : $\int(\nabla\times\mathbf{A})\cdot d\mathbf{a} = \oint\mathbf{A}\cdot d\mathbf{l}$

FUNDAMENTAL CONSTANTS

ϵ_0	$= 8.85 \times 10^{-12}\,C^2/Nm^2$	(permittivity of free space)
μ_0	$= 4\pi \times 10^{-7}\,N/A^2$	(permeability of free space)
c	$= 3.00 \times 10^8\,m/s$	(speed of light)
e	$= 1.60 \times 10^{-19}\,C$	(charge of the electron)
m	$= 9.11 \times 10^{-31}\,kg$	(mass of the electron)

SPHERICAL AND CYLINDRICAL COORDINATES

Spherical

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases} \qquad \begin{cases} \hat{\mathbf{x}} = \sin\theta\cos\phi\,\hat{\mathbf{r}} + \cos\theta\cos\phi\,\hat{\boldsymbol{\theta}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin\theta\sin\phi\,\hat{\mathbf{r}} + \cos\theta\sin\phi\,\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}} \end{cases}$$

$$\begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \theta = \tan^{-1}(\sqrt{x^2+y^2}/z) \\ \phi = \tan^{-1}(y/x) \end{cases} \qquad \begin{cases} \hat{\mathbf{r}} = \sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\,\hat{\mathbf{x}} + \cos\theta\sin\phi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \end{cases}$$

Cylindrical

$$\begin{cases} x = s\cos\phi \\ y = s\sin\phi \\ z = z \end{cases} \qquad \begin{cases} \hat{\mathbf{x}} = \cos\phi\,\hat{\mathbf{s}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin\phi\,\hat{\mathbf{s}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

$$\begin{cases} s = \sqrt{x^2+y^2} \\ \phi = \tan^{-1}(y/x) \\ z = z \end{cases} \qquad \begin{cases} \hat{\mathbf{s}} = \cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}} \\ \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

Standaardintegralen .

$I = \int x^m (a^2+x^2)^n\,dx$ Noem $Y = \sqrt{a^2+x^2}$; $Y^2 = a^2+x^2$;

m	n	I	m	n	I
-2	-1/2	$-Y/(a^2x)$	1	-3/2	$-1/Y$
-2	-1	$-a^{-2}(\frac{1}{x}+\frac{1}{a}\arctan\frac{x}{a})$	1	-1	$\ln\lvert Y\rvert$
-1	-3/2	$a^{-2}(\frac{1}{Y}-\frac{1}{a}\ln\left\lvert\frac{a+Y}{x}\right\rvert)$	1	-1/2	Y
-1	-1/2	$-(1/a)\ln\lvert(a+Y)/x\rvert$	1	1/2	$\frac{1}{3}Y^3$
-1	-1	$a^{-2}\ln\lvert x/Y\rvert$	1	3/2	$\frac{1}{5}Y^5$
0	-3/2	$x/(a^2Y)$	2	-3/2	$\ln\lvert x+Y\rvert - x/Y$
0	-1	$a^{-1}\arctan(x/a)$	2	-1	$x - a\arctan(x/a)$
0	-1/2	$\ln\lvert x+Y\rvert$	2	-1/2	$\frac{1}{2}xY - \frac{1}{2}a^2\ln\lvert x+Y\rvert$
0	1/2	$\frac{1}{2}xY + \frac{1}{2}a^2\ln\lvert x+Y\rvert$	2	1/2	$\frac{1}{8}x(2x^2+a^2)Y - \frac{1}{8}a^4\ln\lvert x+Y\rvert$
0	3/2	$\frac{1}{8}x(2x^2+5a^2)Y + \frac{3}{8}a^4\ln\lvert x+Y\rvert$	3	-3/2	$Y + a^2/Y$
			3	-1/2	$\frac{1}{3}Y^3 - a^2Y$
			3	1/2	$\frac{1}{5}Y^5 - \frac{1}{3}a^2Y^3$

$I = \int \sin^m ax \cos^n ax\,dx$

m	n	I	m	n	I
1	0	$-(1/a)\cos ax$	1	1	$(\sin^2 ax)/2a$ of $-(\cos^2 ax)/2a$
0	1	$(1/a)\sin ax$	2	2	$-\frac{1}{32a}\sin 4ax + \frac{x}{8}$
1	-1	$-(1/a)\ln\lvert\cos ax\rvert$	1	n	$-\frac{\cos^{n+1} ax}{(n+1)a}$
-1	1	$(1/a)\ln\lvert\sin ax\rvert$	m	1	$\frac{\sin^{m+1} ax}{(m+1)a}$
2	0	$\frac{1}{2}x - \frac{1}{4a}\sin 2ax$	0	2	$\frac{1}{2}x + \frac{1}{4a}\sin 2ax$
3	0	$-\frac{1}{3a}\cos ax(\sin^2 ax + 2)$	0	3	$\frac{1}{3a}\sin ax(\cos^2 ax + 2)$
4	0	$\frac{3x}{8} - \frac{\sin 2ax}{4a} + \frac{\sin 4ax}{32a}$	0	4	$\frac{3x}{8} + \frac{\sin 2ax}{4a} + \frac{\sin 4ax}{32a}$

6.5 Benaderingen voor $|x| \to 0$

$(1+x)^a$	$1 + ax + \ldots$	$\sin x$	$x - x^3/6 + \ldots$
e^x	$1 + x + \ldots$	$\cos x$	$1 - x^2/2 + \ldots$
$\ln(1+x)$	[illegible]		