Test 1 for Probability Theory
(Module Signals and Uncertainty, 201300182)
Friday May 10, 2019, 8.45 - 10.15 hour.

This test consists of 4 problems and 1 table (P.T.O.)
Use proper notation and motivate all answers.
Using a calculator is *not* allowed.

1. Given is a probability space (S, P), and the events E and F.
   a. Prove that $P(EF) \geq P(E) + P(F) - 1$.
   b. Is equality possible (i.e., $P(EF) = P(E) + P(F) - 1$) when E and F are independent?
   c. In two sequences of 5 coin flips with a fair coin, what is the probability that both sequences give exactly the same result (in the same order)?
   d. Same question as c., but now *given* that both sequences yield three times heads and two times tails?

2. Let X have a probability mass function $p(i) = (1-p)p^i,\ i = 0, 1, 2, \ldots$.
   a. Show that $p(\cdot)$ is indeed a proper probability mass function.
   b. Let $Y = X + 1$. Determine the probability mass function for Y and show that it corresponds to a geometric distribution.
   c. Determine EX.
   d. A median of X is a value m for which both $P(X \leq m) \geq 0.5$ and $P(X \geq m) \geq 0.5$ hold. Determine the median(s) of X when $p = 0.7$.

3. The random variable X has a (cumulative) distribution function given by $F(x) = 1 - \frac{1}{2}(e^{-x} + e^{-2x}),\ x \geq 0$.
   a. Determine $P(-1 < X < 2)$.
   b. Compute $E[X]$.
   c. Give the probability density function of the random variable Y, where Y is defined as $Y = \sqrt{|X|}$.

4. Let X have a normal distribution with parameters μ and σ^2.
   a. Determine $P(\mu - 2\sigma < X < \mu + \sigma)$.
   b. Show by integration that, for the case $\mu = 0$ and $\sigma^2 = 1$, we have $E[X] = 0$ and $\mathrm{Var}(X) = 1$.

Norm: (Grade = total/3 + 1)

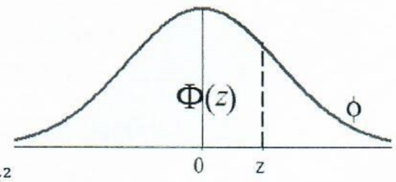
1				2				3			4		Total
a	b	c	d	a	b	c	d	a	b	c	a	b	
2	1	2	2	1	3	2	2	2	2	3	2	3	27

P.T.O.

Standard normal probabilities

The tabel gives the distribution function Φ for a N(0, 1) variable Z

$$\Phi(z) = P(Z \leq z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{x^2}{2}}\, dx$$



The last column the N(0,1) gives the density function: $\varphi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$

	Second decimal of z										
z	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09	φ(z)
0,0	0,5000	0,5040	0,5080	0,5120	0,5160	0,5199	0,5239	0,5279	0,5319	0,5359	0,3989
0,1	0,5398	0,5438	0,5478	0,5517	0,5557	0,5596	0,5636	0,5675	0,5714	0,5753	0,3970
0,2	0,5793	0,5832	0,5871	0,5910	0,5948	0,5987	0,6026	0,6064	0,6103	0,6141	0,3910
0,3	0,6179	0,6217	0,6255	0,6293	0,6331	0,6368	0,6406	0,6443	0,6480	0,6517	0,3814
0,4	0,6554	0,6591	0,6628	0,6664	0,6700	0,6736	0,6772	0,6808	0,6844	0,6879	0,3683
0,5	0,6915	0,6950	0,6985	0,7019	0,7054	0,7088	0,7123	0,7157	0,7190	0,7224	0,3521
0,6	0,7257	0,7291	0,7324	0,7357	0,7389	0,7422	0,7454	0,7486	0,7517	0,7549	0,3332
0,7	0,7580	0,7611	0,7642	0,7673	0,7704	0,7734	0,7764	0,7794	0,7823	0,7852	0,3123
0,8	0,7881	0,7910	0,7939	0,7967	0,7995	0,8023	0,8051	0,8078	0,8106	0,8133	0,2897
0,9	0,8159	0,8186	0,8212	0,8238	0,8264	0,8289	0,8315	0,8340	0,8365	0,8389	0,2661
1,0	0,8413	0,8438	0,8461	0,8485	0,8508	0,8531	0,8554	0,8577	0,8599	0,8621	0,2420
1,1	0,8643	0,8665	0,8686	0,8708	0,8729	0,8749	0,8770	0,8790	0,8810	0,8830	0,2179
1,2	0,8849	0,8869	0,8888	0,8907	0,8925	0,8944	0,8962	0,8980	0,8997	0,9015	0,1942
1,3	0,9032	0,9049	0,9066	0,9082	0,9099	0,9115	0,9131	0,9147	0,9162	0,9177	0,1714
1,4	0,9192	0,9207	0,9222	0,9236	0,9251	0,9265	0,9279	0,9292	0,9306	0,9319	0,1497
1,5	0,9332	0,9345	0,9357	0,9370	0,9382	0,9394	0,9406	0,9418	0,9429	0,9441	0,1295
1,6	0,9452	0,9463	0,9474	0,9484	0,9495	0,9505	0,9515	0,9525	0,9535	0,9545	0,1109
1,7	0,9554	0,9564	0,9573	0,9582	0,9591	0,9599	0,9608	0,9616	0,9625	0,9633	0,0940
1,8	0,9641	0,9649	0,9656	0,9664	0,9671	0,9678	0,9686	0,9693	0,9699	0,9706	0,0790
1,9	0,9713	0,9719	0,9726	0,9732	0,9738	0,9744	0,9750	0,9756	0,9761	0,9767	0,0656
2,0	0,9772	0,9778	0,9783	0,9788	0,9793	0,9798	0,9803	0,9808	0,9812	0,9817	0,0540
2,1	0,9821	0,9826	0,9830	0,9834	0,9838	0,9842	0,9846	0,9850	0,9854	0,9857	0,0440
2,2	0,9861	0,9864	0,9868	0,9871	0,9875	0,9878	0,9881	0,9884	0,9887	0,9890	0,0355
2,3	0,9893	0,9896	0,9898	0,9901	0,9904	0,9906	0,9909	0,9911	0,9913	0,9916	0,0283
2,4	0,9918	0,9920	0,9922	0,9925	0,9927	0,9929	0,9931	0,9932	0,9934	0,9936	0,0224
2,5	0,9938	0,9940	0,9941	0,9943	0,9945	0,9946	0,9948	0,9949	0,9951	0,9952	0,0175
2,6	0,9953	0,9955	0,9956	0,9957	0,9959	0,9960	0,9961	0,9962	0,9963	0,9964	0,0136
2,7	0,9965	0,9966	0,9967	0,9968	0,9969	0,9970	0,9971	0,9972	0,9973	0,9974	0,0104
2,8	0,9974	0,9975	0,9976	0,9977	0,9977	0,9978	0,9979	0,9979	0,9980	0,9981	0,0079
2,9	0,9981	0,9982	0,9982	0,9983	0,9984	0,9984	0,9985	0,9985	0,9986	0,9986	0,0060
3,0	0,9987	0,9987	0,9987	0,9988	0,9988	0,9989	0,9989	0,9989	0,9990	0,9990	0,0044
3,1	0,9990	0,9991	0,9991	0,9991	0,9992	0,9992	0,9992	0,9992	0,9993	0,9993	0,0033
3,2	0,9993	0,9993	0,9994	0,9994	0,9994	0,9994	0,9994	0,9995	0,9995	0,9995	0,0024
3,3	0,9995	0,9995	0,9995	0,9996	0,9996	0,9996	0,9996	0,9996	0,9996	0,9997	0,0017
3,4	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9998	0,0012
3,5	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,0009
3,6	0,9998	0,9998	0,9999	0,9999	0,9999	0,9999	0,9999	0,9999	0,9999	0,9999	0,0006