Test 1 for Probability Theory
(Module Signals and Uncertainty, 201300182)
Friday February 23, 2018, 8.45 - 10.15 hour.

This test consists of 4 problems and 1 table (P.T.O.)
Use proper notation and motivate all answers.
Using a calculator is *not* allowed.

1. Given is a probability space (S, P), and some fixed event $A \subset S$ with $P(A) > 0$. Define $P(\cdot|A)$ as usual, i.e. for each event $B \subset S$ the value of $P(B|A)$ is just the conditional probability of B given A.

   a. Prove that $P(\cdot|A)$ is a proper probability measure on S, by stating and checking the Kolmogorov axioms.

   Now consider a fair coin and an unfair coin, where the latter has probability $1/4$ of coming up heads.

   b. We select one of the coins at random and then flip it until it comes up heads. If this turns out to happen at the third flip, then what is the probability that we selected the fair coin?

   c. In another experiment with the same two coins, we again select one at random, and then flip it 3 times. What is the probability that heads comes up exactly 2 times?

2. The random variable X has the following probability mass function,

$$p(i) = e^{-\lambda}\frac{\lambda^{i-1}}{(i-1)!}, \quad i = 1, 2, \ldots$$

   a. Give the range of X and show that $p(\cdot)$ is indeed a proper probability mass function.

   b. Determine the expectation of X.

   c. Find $P(Y < 1)$, where the random variable Y is defined as $Y = |X-3|$.

3. For a fixed parameter $a > 0$, the random variable X has a uniform distribution on the interval $[-a, a]$.

   a. Find the variance of X.

   b. Give the (cumulative) distribution function $F_X(x)$ of X for all x.

   c. Give the probability density function of the random variable Y, where Y is defined as $Y = X^2$.

4. The random variable X has a normal distribution with parameters $\mu = 1$ and $\sigma^2 = 4$. Determine $P(X^2 < 2 - X)$.

Norm: (Grade = total/3 + 1)

1			2			3			4	Total
a	b	c	a	b	c	a	b	c		
5	3	2	2	3	2	2	2	3	3	27