Test 1 for Probability Theory
(Module Signals and Uncertainty, 202001342)
Monday May 16, 2022, 8.45 - 10.15 hour.

This test consists of 4 problems and 1 table (P.T.O.)
Use proper notation and motivate all answers.
Using a calculator is *not* allowed.

1. Consider a probability space (S, P), and the events E and F.
   a. Using (only) the three axioms of Kolmogorov, prove that $E \subset F$ implies $P(E) \leq P(F)$.
   b. Suppose that E is a strict subset of F, i.e., $E \subset F$ and $E \neq F$. Can you conclude that $P(E) < P(F)$? Why (not)?
   c. A fair coin was flipped five times. Compute the probability that the coin came up heads during the first and second time it was flipped, when we are told that the total number of heads was three.

2. We roll a single fair dice until a five comes up. Let X be the total number of rolls needed.
   a. Give the range S_X and the probability mass function $p(i), i \in S_X$.
   b. Determine $P(X > 2)$.
   c. Let Y be the number of rolls *before* the first five comes up, i.e., $Y = X - 1$. Determine EY.
   d. Sophie claims that $Var(Y) = Var(X)$, while Dave claims that $Var(Y) < Var(X)$. Explain who is right and why, or explain why you think both are wrong.

3. A continuous random variable X has a probability density given by $f(x) = \frac{2}{3}(e^{-x} + e^{-2x})$, $x \geq 0$ (and $f(x) = 0$ if $x < 0$).
   a. Determine the distribution function of X.
   b. Determine EX.
   c. Determine the density of the random variable Y given by $Y = X + 5$.

4. Let X have a normal distribution with expectation 0 and variance $(2\pi)^{-1}$.
   a. Give the (approximate) value of $P(100X^2 > 2\pi)$.
   b. What can you say about the probability distribution of $Y = 3X + 4$?

Norm: (Grade = total/3 + 1)

1			2				3			4		Total
a	b	c	a	b	c	d	a	b	c	a	b	
2	2	3	2	2	2	2	2	2	3	3	2	27

P.T.O.