1 $P(G) = P(O) = \frac{1}{2}$

a. $P(K|G) = \frac{1}{2} \quad P(K|O) = 1$

$P(K) = P(K|G)P(G) + P(K|O)P(O) = \frac{3}{4}$

$P(G|K) = P(K|G)P(G)/P(K) = \frac{1}{3}$

b $P(KK|G) = \frac{1}{4} \quad P(KK|O) = 1$

$P(KK) = P(KK|G)P(G) + P(KK|O)P(O) = \frac{5}{8}$

$P(G|KK) = P(KK|G)P(G)/P(KK) = \frac{1}{5}$

c $P(G|KKM) = 1$

2 a

$$c \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{2^i}{i!} \frac{3^j}{j!} = ce^5 = 1 \quad \Rightarrow \quad c = e^{-5}$$

b

$$P(X+Y=1) = P(X=0, Y=1) + P(X=1, Y=0)$$

$$= 5c = 5e^{-5}$$

c

$$P(X=i) = \sum_{j=0}^{\infty} P(X=i, Y=j) = e^{-2} \frac{2^i}{i!}, \quad i = 0, 1, \ldots$$

$EX = 2 \quad EX^2 = 6 \quad \operatorname{var} X = 2$

Poissonverdeling

d ja, want $P(X=i, Y=j) = P(X=i)P(Y=j) \quad i, j = 0, 1, \ldots$

3 a. $f_X(x) = \begin{cases} \frac{1}{\pi} & 0 < x < \pi \\ 0 & \text{elders} \end{cases}$

$$\varphi_X(t) = E[e^{tX}] = \int_0^\pi \frac{1}{\pi} e^{tx}\, dx$$

$$= \frac{1}{\pi t}\left(e^{\pi t} - 1\right)$$

b $$E[\sin X] = \int_0^\pi \sin x \frac{1}{\pi}\, dx$$

$$= -\frac{1}{\pi}\cos x \Big|_0^\pi = \frac{2}{\pi}$$

c $$F_Y(x) = P(Y \leq x) = P(X^2 \leq x)$$

$$= \begin{cases} 0 & x \leq 0 \\ \frac{1}{\pi}\sqrt{x} & 0 < x < \pi^2 \\ 1 & x \geq \pi^2 \end{cases}$$

dus

$$f_Y(x) = \begin{cases} \frac{1}{2\pi\sqrt{x}} & 0 < x < \pi^2 \\ 0 & \text{elders} \end{cases}$$

4 a $$EX^2 = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx = \int_0^1 x^3 dx + \int_1^2 x^2(2-x)\, dx$$

$$= \frac{1}{4} + \frac{11}{12} = \frac{7}{6} \qquad EX = 1$$

$$\operatorname{var} X = EX^2 - (EX)^2 = \frac{7}{6} - 1 = \frac{1}{6}$$

c $$ES_n = n\, EX_1 = n$$

$$\operatorname{var} S_n = n \operatorname{var} X_1 = \frac{1}{6} n$$

4 b $\quad \mathrm{cov}(X_1, S_2) = \mathrm{cov}(X_1, X_1 + X_2)$

$= \mathrm{cov}(X_1, X_1) + \mathrm{cov}(X_1, X_2)$

$= \mathrm{var}(X_1) = \frac{1}{6}$

4 d $\quad$ C.L.S. : $\quad \lim_{n\to\infty} P\left(\frac{S_n - ES_n}{\sqrt{\mathrm{var} S_n}} \leq x \right) = \Phi(x)$

$\iff \lim_{n\to\infty} P\left(\frac{S_n - n}{\sqrt{n/6}} \leq x \right) = \Phi(x)$

$\iff \lim_{n\to\infty} P\left(\frac{S_n - n}{\sqrt{n}} \leq \frac{x}{\sqrt{6}} \right) = \Phi(x)$

dus

$$\lim_{n\to\infty} P\left(\frac{S_n - n}{\sqrt{n}} \leq 1 \right) = \Phi(\sqrt{6})$$