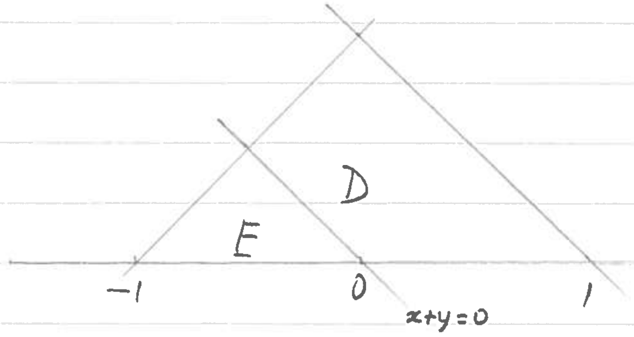
1.



a. $$f(x,y) = \begin{cases} 1 & (x,y) \in D \\ 0 & \text{elders} \end{cases}$$

b $$P(X+Y \leq 0) = P((X,Y) \in E) = \tfrac{1}{4}$$

c $$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \begin{cases} 1-|x| & |x|<1 \\ 0 & \text{elders} \end{cases}$$

$$E(|X|) = \int_{-1}^{1} |x|(1-|x|)\,dx = 2\int_0^1 x(1-x)\,dx = \tfrac{1}{3}$$

d $f_{Y|X}(y|x)$ is gedefinieerd voor $|x|<1$, dan

$$f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} = \begin{cases} \frac{1}{1-|x|} & 0 \leq y < 1-|x| \\ 0 & \text{elders} \end{cases}$$

e

$$E(Y|X=x) = \int_{-\infty}^{\infty} y\, f_{Y|X}(y|x)\,dy$$

$$= \int_0^{1-|x|} y\,\frac{1}{1-|x|}\,dy = \tfrac{1}{2}(1-|x|)$$

$$E(Y|X) = \tfrac{1}{2}(1-|X|)$$

$$EY = E(E(Y|X)) = E(\tfrac{1}{2}(1-|X|)) = \tfrac{1}{2} - \tfrac{1}{2}E|X| = \tfrac{1}{3}$$

2 a $E(N_{j+1} | N_j = m) = pm$ want $(N_{j+1} | N_j = m) \sim B(m, p)$

$E(N_{j+1} | N_j) = p N_j$.

b $E(N_j) = E(E(N_j | N_{j-1})) = E(p N_{j-1}) = p E N_{j-1}$

$\therefore E(N_j) = p^{j-1} E(N_1) = p^j n$ want $N_1 \sim B(n, p)$

c

$$E(e^{tN_{j+1}}) = E(E(e^{tN_{j+1}} | N_j))$$

$$= E\left((pe^t + (1-p))^{N_j} \right)$$

als $N_j \sim B(p^j, n)$:

$$= \left(p^j (pe^t + (1-p)) + (1-p^j) \right)^n$$

$$= \left(p^{j+1} e^t + 1 - p^{j+1} \right)^n$$

dus $N_{j+1} \sim B(p^{j+1}, n)$.

óf

0	0	1	1	0	1	0	1	0	1	$N_1 = 5$
		1	0		0		1		1	$N_2 = 3$
		1					0		1	$N_3 = 2$

$P(N_j = m) = P($ j keer achter elkaar succes (1) op n posities $)$

$\therefore N_j \sim B(n, p^j)$

3 a $P(T \leq t \mid M=1) = P(\min(X_1, X_2) \leq t)$

$= 1 - P(X_1 > t)P(X_2 > t) = 1 - e^{-\lambda_1 t} e^{-\lambda_2 t}, \quad t \geq 0$

$\therefore \; f_{T|M}(t|1) = \frac{d}{dt}\left(1 - e^{-(\lambda_1+\lambda_2)t}\right) = (\lambda_1+\lambda_2)e^{-(\lambda_1+\lambda_2)t}, \quad t \geq 0$

b

$E(T \mid M=1, T>1) = 1 + E(T \mid M=1) = 1 + \frac{1}{\lambda_1+\lambda_2}$

c

$E(T \mid M=1) = E(T \mid M=1, T \leq 1)P(T \leq 1) + E(T \mid M=1, T>1)P(T>1)$

$\therefore \; \frac{1}{\lambda_1+\lambda_2} = E(T \mid M=1, T \leq 1)\left(1 - e^{-(\lambda_1+\lambda_2)}\right) + \left(1 + \frac{1}{\lambda_1+\lambda_2}\right)e^{-(\lambda_1+\lambda_2)}$

$\therefore \; E(T \mid M=1, T \leq 1) = \frac{1}{\lambda_1+\lambda_2} - \frac{1}{e^{\lambda_1+\lambda_2} - 1}$

d

$P(M=i) = \frac{\binom{2}{i}\binom{3}{2-i}}{\binom{5}{2}} = \begin{cases} 0.3 & i=0 \\ 0.6 & 1 \\ 0.1 & 2 \end{cases}$

of

$P(M=0) = \frac{3}{5} \cdot \frac{2}{4} \quad P(M=2) = \frac{2}{5} \cdot \frac{1}{4} \quad \Rightarrow P(M=1) = \frac{6}{10}$

e $P(T \leq t) = \frac{3}{10} P(T \leq t \mid M=0) + \frac{6}{10} P(T \leq t \mid M=1) + \frac{1}{10} P(T \leq t \mid M=2)$

$= \frac{3}{10}\left(1 - e^{-2\lambda_2 t}\right) + \frac{6}{10}\left(1 - e^{-(\lambda_1+\lambda_2)t}\right) + \frac{1}{10}\left(1 - e^{-2\lambda_1 t}\right)$

$= 1 - \frac{3}{10}e^{-2\lambda_2 t} - \frac{6}{10}e^{-(\lambda_1+\lambda_2)t} - \frac{1}{10}e^{-2\lambda_1 t}, \quad t \geq 0$

$f_T(t) = \frac{3}{5}\lambda_2 e^{-2\lambda_2 t} + \frac{3}{5}(\lambda_1+\lambda_2)e^{-(\lambda_1+\lambda_2)t} + \frac{1}{5}\lambda_1 e^{-2\lambda_1 t}$

$, \; t \geq 0$