1 a. L = "overlijdt aan longkanker"
R = "roker"

$P(L|R) = 0.18 \quad P(L|\bar{R}) = 0.01 \quad P(R) = 0.3$

b $P(L) = P(L|R)P(R) + P(L|\bar{R})P(\bar{R}) = 0.061$

c $P(R|L) = \dfrac{P(L|R)P(R)}{P(L)} = \dfrac{0.18 \cdot 0.3}{0.061} = \dfrac{54}{61} = 0.885$

d $P(\bar{R} \cap L) = P(L|\bar{R})P(\bar{R}) = 0.007$

2 a. $P(X=k) = (1-p)^{k-1} p$, $k = 1, 2, ..$ met $p = \frac{1}{6}$

$EX = \sum_{k=1}^{\infty} k(1-p)^{k-1} p = \frac{1}{p} = 6$

b.
$$P(X=i, Y=j) = \begin{cases} \left(\frac{2}{3}\right)^{i-1} \frac{1}{6} \left(\frac{5}{6}\right)^{j-i-1} \frac{1}{6} & i<j \\ \left(\frac{2}{3}\right)^{j-1} \frac{1}{6} \left(\frac{5}{6}\right)^{i-j-1} \frac{1}{6} & i>j \end{cases}$$

$$= \begin{cases} \frac{1}{20} \left(\frac{5}{6}\right)^{j} \left(\frac{4}{5}\right)^{i} & i<j \\ \frac{1}{20} \left(\frac{5}{6}\right)^{i} \left(\frac{4}{5}\right)^{j} & i>j \end{cases}$$

c. $P(X=1, Y=1) = 0 \neq \frac{1}{36} = P(X=1)P(Y=1)$

dus X en Y niet o.o.

3 a. $\varphi_X(t) = Ee^{tX} = \frac{1}{2}\int_{-1}^{1} e^{tx}\,dx = \frac{1}{2t}(e^{t}-e^{-t})$

b. $F_Y(y) = P(Y \leq y) = P(X^2 \leq y)$

$$= \begin{cases} 0 & y<0 \\ P(-\sqrt{y} < X < \sqrt{y}) = \sqrt{y} & 0 \leq y \leq 1 \\ 1 & y>1 \end{cases}$$

$$f_Y(y) = \begin{cases} \frac{1}{2\sqrt{y}}\,, & 0 \leq y \leq 1 \\ 0 & \text{elders} \end{cases}$$

c. $EY = EX^2 = \frac{1}{2}\int_{-1}^{1} x^2\,dx = \frac{1}{3}$

$EY^2 = EX^4 = \frac{1}{2}\int_{-1}^{1} x^4\,dx = \frac{1}{5}$

$\operatorname{var} Y = \frac{1}{5} - \left(\frac{1}{3}\right)^2 = \frac{4}{45}$

d $EX^3 = \frac{1}{2}\int_{-1}^{1} x^3\,dx = 0$ $\qquad EX = \frac{1}{2}\int_{-1}^{1} x\,dx = 0$

$\operatorname{cov}(X,Y) = \operatorname{cov}(X,X^2) = EX^3 - EX\,EX^2 = 0$

4 a $EL = 25.8 = 200$ $\qquad \operatorname{var} L = 25.4 = 100$

b $P(L > 190) = P\left(\frac{L-EL}{\sqrt{\operatorname{var} L}} > \frac{190-EL}{\sqrt{\operatorname{var} L}}\right) \approx 1 - \Phi\left(\frac{190-EL}{\sqrt{\operatorname{var} L}}\right)$

$\therefore\ P(L > 190) \approx 1 - \Phi(-1) = \Phi(1) = 0{,}8413$

c $P(L > n) \geq 0{,}9$

$\iff P\left(\frac{L-EL}{\sqrt{\operatorname{var} L}} > \frac{n-EL}{\sqrt{\operatorname{var} L}}\right) \approx \Phi\left(\frac{EL-n}{\sqrt{\operatorname{var} L}}\right) \geq 0{,}9$

$\iff \frac{200-n}{10} \geq 1{,}282 \iff n \leq 187$