UNIVERSITY OF TWENTE

Department of Electrical Engineering, Mathematics and Computer Science

Exam Signals and Transforms on Thursday March 30, 2017, 8.45 – 10.15 hours.

The solutions of the exercises should be clearly formulated. Moreover, in all cases you should motivate your answer!

You are not allowed to use a calculator. Besides pen and paper, the only thing you are allowed to use is one handwritten, singe-sided, A4-sized page of personal notes.

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1. Let $f(t)$ be the 4-periodic function which satisfies:

$$f(t) = t\,\mathrm{rect}(t), \qquad \text{for } t \in [-2,2)$$

a) Sketch the function $f(t)$ for $t \in [-5,5]$.

b) Show that the complex Fourier coefficients are equal to:

$$f_{2m+1} = -\frac{2i}{(2m+1)^2\pi^2}(-1)^m$$

$$f_{2m} = \begin{cases} \frac{i}{2m\pi}(-1)^m & m \neq 0 \\ 0 & m = 0 \end{cases}$$

Something went wrong!

c) Determine the real Fourier series for $f(t)$.

d) Determine the generalized derivative of the function f.

2. Determine the convolution of $f(t) = \mathrm{rect}_2(t+1)$ and $g(t) = \sin(\pi t)\,\mathbb{1}(t)$ **without** the use of Fourier or Laplace transformations.

3. Consider the space $C[-1,1]$ of continuous functions on the interval $[-1,1]$.

a) Consider the sequence of functions $\{f_1, f_2, \ldots\}$ with:

$$f_n(t) = n^{1/3}t^n$$

Verify whether this sequence of functions form a Cauchy sequence or not.

Consider the linear mapping $\mathcal{A} : C[-1,1] \to C[-1,1]$ with $\mathcal{A}(f) = g$ where

$$g(t) = \mathbb{1}(t)f(t)$$

b) Determine the null space of $\mathcal{A}$. In other words, determine $\ker(\mathcal{A})$.

c) Determine $\|\mathcal{A}\|$.

For the exercises the following number of points can be obtained:

Exercise 1. 11 points Exercise 2. 6 points Exercise 3. 10 points

The grade is determined by adding 3 points to the total number of points obtained and dividing by 3.