UNIVERSITY OF TWENTE

Department of Electrical Engineering, Mathematics and Computer Science

Exam Signals and Transforms on Tuesday March 27, 2018, 8.45 - 10.15 hours.

The solutions of the exercises should be clearly formulated. Moreover, in all cases you should motivate your answer!

You are not allowed to use a calculator. Besides pen and paper, the only thing you are allowed to use is one handwritten, singe-sided, A4-sized page of personal notes.

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1. Consider a filter for which the step response is given by:

$$g(t) = \cos(\pi t)\,\mathrm{trian}_2(t)$$

   a) Determine the impulse response $h(t)$

   b) Show the frequency response is given by:

$$\hat{h}(\omega) = \frac{2i\omega(\omega^2+\pi^2)\sin^2\omega}{(\omega-\pi)^2(\omega+\pi)^2}$$

   c) Consider the input:

$$u(t) = \mathrm{sgn}(t) = \mathbb{1}(t) - \mathbb{1}(-t)$$

   Show that the corresponding output is given by $y(t) = 2g(t)$.

   d) Consider the input:

$$u(t) = \sin\left(\tfrac{\pi}{2}t\right)$$

   Compute the corresponding output $y(t)$ and show it is real-valued.

2. Determine the convolution of $f(t) = \mathrm{rect}_2(t)$ and $g(t) = e^t\,\mathbb{1}(1-t)$ via Fourier or Laplace transformation.

3. Given is the differential equation:

$$y^{(2)}(t) + 6y^{(1)}(t) + 9y(t) = u(t). \tag{1}$$

   a) Determine the impulse response of (1).

   b) Determine the step response of (1).

   c) As input we choose $u(t) = e^{-2t}\,\mathbb{1}(t)$. Determine the solution for $t > 0$ of (1) with $y(0^-) = 2$, $y'(0^-) = -6$.

For the exercises the following number of points can be obtained:

Exercise 1. 10 points Exercise 2. 7 points Exercise 3. 10 points

The grade is determined by adding 3 points to the total number of points obtained and dividing by 3.