Signals & Transforms — TEST 1 (resit)
(part of AM module 4)

Date:	06-06-2019
Place:	OH 215
Time:	8:45–10:15 (till 10:45 for students with special rights)
Course coordinator:	G. Meinsma
Allowed aids during test:	NONE

The solutions of the exercises should be clearly formulated. Moreover, in all cases you should motivate your answer! You are not allowed to use a calculator.

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1. Consider the function $f(t)$ with period $T = 2$ which equals

$$f(t) = |t| \qquad \text{for } t \in [-1, 1).$$

   (a) Sketch the function $f(t)$ for $t \in [-3, 3]$.

   (b) Show that the real Fourier series of $f(t)$ is

$$\frac{1}{2} + \sum_{m=0}^{\infty} \frac{-4}{(2m+1)^2\pi^2} \cos((2m+1)\pi t).$$

   (c) Determine the complex Fourier coefficients f_k.

   (d) Use the above to compute $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$.

2. Determine the convolution of $f(t) = \mathrm{rect}_{2\pi}(t) + \delta(t-1)$ and $g(t) = \sin(t)\mathbb{1}(t)$ **without the use of Fourier or Laplace transformations.**

3. Give the definition of *inner product* on a complex vector space.

4. Let $T > 0$ and consider the linear mapping $A : \mathscr{L}^2([0, T]; \mathbb{C}) \to \ell^2(\mathbb{Z}; \mathbb{C})$ with $A(f) = g$ defined as

$$g(k) = \frac{1}{T}\int_0^T f(t) \mathrm{e}^{-\mathrm{i}k\frac{2\pi}{T}t}\,\mathrm{d}t, \qquad k \in \mathbb{Z}.$$

   On $\mathscr{L}^2([0, T]; \mathbb{C})$ we take the standard norm

$$\|f\| = \sqrt{\int_0^T |f(t)|^2\,\mathrm{d}t},$$

   and on $\ell^2(\mathbb{Z}; \mathbb{C})$ we take the standard norm

$$\|g\| = \sqrt{\sum_{k=-\infty}^{\infty} |g(k)|^2}.$$

   Determine the operator norm $\|A\|$.

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problem:	1	2	3	4
points:	1+6+3+2	7	3	5

Test grade is $1 + 9p/p_{\max}$

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Property		Condition
Sifting	$\int_{-\infty}^{\infty} \delta(t-b) f(t)\, \mathrm{d}t = f(b)$	$f(t)$ continuous at $t = b$
-	$f(t)\delta(t-b) = f(b)\delta(t-b)$	$f(t)$ continuous at $t = b$
Convolution	$(f * \delta)(t) = f(t)$	
Scaling	$\delta(at-b) = \frac{1}{\lvert a\rvert}\delta(t-\frac{b}{a})$	
-	$\int_{-\infty}^{t} \delta(\tau)\, \mathrm{d}\tau = \mathbb{1}(t)$	$t \neq 0$

Property	Time domain: $f(t)$	Frequency domain: f_k
Linearity	$\alpha f(t) + \beta g(t)$	$\alpha f_k + \beta g_k$
Time-shift	$f(t-\tau), \quad (\tau \in \mathbb{R})$	$\mathrm{e}^{-ik\omega_0\tau} f_k$
Time-reversal	$f(-t)$	f_{-k}
Conjugation	$f^*(t)$	f^*_{-k}
Frequency-shift	$\mathrm{e}^{in\omega_0 t} f(t), \quad (n \in \mathbb{Z})$	f_{k-n}