Signals & Transforms — TEST 1
(part of AM module 4 — 201800138)

Date:	13-05-2020
Place:	At home!
Time:	13:45–15:15 (till 15:40 for students with special rights)
Course coordinator:	G. Meinsma
Allowed aids during test:	see below

The solutions of the exercises should be clearly formulated. Moreover, in all cases you should motivate your answer! You are not allowed to use a calculator.

Integrity statement Please read the following paragraph carefully.

By testing you remotely in this fashion, we express our trust that you will adhere to the ethical standard of behaviour expected of you. This means that we trust you to answer the questions and perform the assignments in this test to the best of your own ability, without seeking or accepting the help of any source that is not explicitly allowed by the conditions of this test.

The only allowed sources for this test are:

- the lecture notes "Signals and Transforms" (pdf or printed)
- the slides (pdf or printed)
- electronic devices, but only to be used:
  - for downloading the test and afterwards uploading your work to Canvas
  - to show the test/book/slides on your screen
  - to write the test (in case you prefer to use a tablet instead of paper to write on)

problem:	1	2	3	4
points:	2+5+3+1	6	2	4+4

Test grade is $1 + p/3$

P.T.O.

A. **Copy the following text verbatim to the first page of your work (handwritten) and sign it. If you fail to do so, your test will not be graded:**

> *I will make this test to the best of my own ability, without seeking or accepting the help of any source not explicitly allowed by the conditions of the test.*

B. **What programme do you follow (AM, AM+AP, AM+TCS, Minor, ??)**

C. **Are you entitled to extra time? (We will check this with CES.)**

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1. Consider a sequence of periodic functions $f_n(t)$, all having period 2, and equal to

$$f_n(t) = t^n \qquad \forall -1 < t \leq 1.$$

Denote the kth Fourier coefficient of $f_n(t)$ as $f_{n,k}$.

(a) Let n be a nonnegative integer. Determine $f_{n,0}$.

(b) Let $n \in \mathbb{N}$ (so $n > 0$). Show that

$$f_{n,k} = \frac{1}{\mathrm{i}k\pi} \times \begin{cases} n f_{n-1,k} & \text{if } n \text{ is even, } k \neq 0, \\ n f_{n-1,k} - (-1)^k & \text{if } n \text{ is odd, } k \neq 0. \end{cases}$$

(c) Determine the **real** Fourier series of $f_1(t)$.

(d) For which $n \in \mathbb{N}$ is the Fourier series of $f_n(t)$ equal to $f_n(t)$ for all $t \in \mathbb{R}$?

2. Determine the convolution of $f(t) = \mathbb{1}(-t+1) + \delta(2t+4)$ and $g(t) = \mathrm{e}^t\, \mathbb{1}(-t)$.

3. Consider $\mathbb{R}^2$ with norm $\|x\|_s = |x_1| + |x_1 - x_2|$. Is there an inner product on $\mathbb{R}^2$ such that $\|x\|_s = \sqrt{\langle x, x\rangle}$?

4. Let $\mathbb{X}$ denote the vector space of continuous functions $f : [0,1] \to \mathbb{R}$ with inner product

$$\langle f, g\rangle = \int_0^1 t f(t) g(t)\, \mathrm{d}t.$$

(a) Show that this is a real inner product on $\mathbb{X}$.

(b) In this inner product, determine the best approximation of $f(t) = t$ in the subspace spanned by the two functions 1 and t^2.

P.T.O.

Property		Condition
Sifting	$\int_{-\infty}^{\infty} \delta(t-b) f(t)\,\mathrm{d}t = f(b)$	$f(t)$ continuous at $t = b$
-	$f(t)\delta(t-b) = f(b)\delta(t-b)$	$f(t)$ continuous at $t = b$
Convolution	$(f * \delta)(t) = f(t)$	
Scaling	$\delta(at-b) = \frac{1}{\lvert a \rvert}\delta(t-\frac{b}{a})$	
-	$\int_{-\infty}^{t} \delta(\tau)\,\mathrm{d}\tau = \mathbb{1}(t)$	$t \neq 0$

Property	Time domain: $f(t)$	Frequency domain: f_k
Linearity	$\alpha f(t) + \beta g(t)$	$\alpha f_k + \beta g_k$
Time-shift	$f(t-\tau), \quad (\tau \in \mathbb{R})$	$\mathrm{e}^{-\mathrm{i}k\omega_0\tau} f_k$
Time-reversal	$f(-t)$	f_{-k}
Conjugation	$f^*(t)$	f^*_{-k}
Frequency-shift	$\mathrm{e}^{in\omega_0 t} f(t), \quad (n \in \mathbb{Z})$	f_{k-n}