(Class) Test-1: Analysis II

Statistics and Analysis (201800139)

25-september-2018 , 08:45 – 10:15 Total Points : 22

All answers must be motivated.
Approach to a solution is equally (if not more) important.
Use of an electronic calculator or a book is not allowed.
Good Luck!

1. Determine the convergence/divergence of the following series. In case a series converges, determine whether it also converges absolutely. [2+2]

   (a.) $\displaystyle\sum_{k=1}^{\infty} \frac{2^k - \sqrt{k}}{k!}$

   (b.) $\displaystyle\sum_{k=1}^{\infty} (-1)^k \frac{k}{1+k^2}$

2. (a.) Give the definition of uniform convergence of a series of real-valued functions, using ϵ-δ-N arguments/language. [2]

   (b.) Show that the function $\displaystyle f(x) := \sum_{n=1}^{\infty} \frac{\sin(nx) + \sqrt{n}}{n^2 + x^2}$ is continuous on $\mathbb{R}$. [3]

   (c.) Suppose a sequence of real-valued functions f_n converges uniformly on the closed interval $[a,b]$, with $b > a$. Show that if each f_n is integrable on $[a,b]$, then the limit function is also integrable and satisfies [5]

   $$\lim_{n\to\infty} \int_a^b f_n(x)\,dx = \int_a^b \lim_{n\to\infty} f_n(x)\,dx.$$

3. (a.) Consider the function: $f(x) = x\ln(x) - x$. Show that the Taylor series of $f(x)$ around $x_0 = 1$ is given by [3]

   $$\sum_{k=2}^{\infty} \frac{(-1)^k}{k(k-1)} (x-1)^k \; - \; 1.$$

   (b.) By considering the Taylor series in (a.) as a power series, determine its radius of convergence and its interval of convergence. [3]

   (c.) For which values of x, does the Taylor series in (a.) converge uniformly? Absolutely? Motivate your answer. [1+1]

Grade:	$\dfrac{\text{score on test}}{22} \times 9 + 1$	(rounded off to two decimal places)