(Class) Test-1: Analysis II
Statistics and Analysis (201800139)

24-september-2019, 08:45 – 10:15

Total Points : 21

All answers must be motivated.
Approach to a solution is equally important as the final answer.
Use of an electronic calculator or a book is not allowed.
Good Luck!

1. (a.) Give the definition of absolute convergence of a series of real numbers, using ϵ-δ-N arguments/language. [2]

   (b.) Determine the convergence/divergence of the following series. In case a series converges, determine whether it also converges absolutely. [2+2]

   (i.) $\displaystyle\sum_{k=1}^{\infty}\left(1-\frac{1}{k}\right)^{k}$ **(ii.)** $\displaystyle\sum_{k=1}^{\infty}\left(\frac{3k-\cos(k)}{2k+\sqrt{k}}\right)^{k}$

   (c.) Prove the following part of the "Limit Comaprison Test": [3]
   Suppose $a_n \geq 0,\ b_n > 0,\ \forall n \in \mathbb{N}$.

   If $\displaystyle\lim_{n\to\infty}\frac{a_n}{b_n} = \infty$ and $\displaystyle\sum_{k=1}^{\infty} b_k$ diverges, then $\displaystyle\sum_{k=1}^{\infty} a_k$ diverges as well.

2. Consider the series of real valued functions given by: $\displaystyle\sum_{k=1}^{\infty} a_k \frac{1}{k^x}$, where $a_k \in \mathbb{R},\ \forall k \in \mathbb{N}$.
   Suppose the series converges at $x = x_0$.

   (a.) Show that the the series converges absolutely on (x_0+1, ∞). [3]

   [Hint: First argue, and then use, that the sequence $\dfrac{|a_k|}{k^{x_0}}$ is bounded.]

   (b.) Show that the series converges uniformly on $[x_0+r, \infty)$, for any $r > 1$. [3]

3. (a.) Find the radius of convergence and the convergence interval of the power series: [3]

$$\sum_{k=2}^{\infty} k(k-1)\, x^k.$$

   (b.) Determine a simpler (more compact) form/expression for the power series in 3(a.) on its interval of convergence. [3]

Grade: $\dfrac{\text{score on test}}{21} \times 9 + 1$ (rounded off to two decimal places)