(Class) Test-1: Analysis II
Statistics and Analysis (201800139)

22-september-2020, 10:00 – 11:30 Total Points : 20

All answers must be motivated.
Approach to a solution is equally important as the final answer.
Use of an electronic calculator or a book is not allowed.
Good Luck!

1. (a) Give the definition of conditional convergence of a series of real numbers. [2]

   (b) Determine the convergence/divergence of the following series. In case a series converges, determine whether it also converges absolutely. [2+2]

   $$\text{(i.)} \quad \sum_{k=1}^{\infty}(-1)^k \frac{k^2+\pi}{k^3+\sin(k)} \qquad\qquad \text{(ii.)} \quad \sum_{k=1}^{\infty}\left(\frac{k-\cos(k^2)}{3k+\log(k+1)}\right)^k$$

   (c) Prove the following special "Limit Comparison Test": [3]

   Suppose $a_n \geq 0, \forall n \in \mathbb{N}$ and $b_n \neq 0, \forall n \in \mathbb{N}$. If $\lim_{n\to\infty} \frac{a_n}{b_n} = -1$ and $\sum_{k=1}^{\infty} a_k$ converges, then $\sum_{k=1}^{\infty} b_k$ converges as well.

2. Consider the series of real valued functions given by:

   $$\sum_{k=1}^{\infty} \frac{x}{k} \sin\left(\frac{x}{k}\right).$$

   (a) Show that the series converges uniformly on every bounded interval of $\mathbb{R}$ [3]

   (b) Show that the series converges to a twice continuously differentiable function. [3]

3. (a) Find the radius of convergence and the convergence interval of the power series: [3]

   $$\sum_{k=0}^{\infty}\left(\frac{1}{(-1)^k-5}\right)^k x^k.$$

   (b) Does the power series in 3.(a) converges absolutely at $x=-4.5$? [1]

   (c) What is the radius of convergence of the power series: [1]

   $$\sum_{k=1}^{\infty} k\left(\frac{1}{(-1)^k-5}\right)^k x^{k-1}.$$

Grade:	$\frac{\text{score on test}}{20} \times 9 + 1$ (rounded off to one decimal place)