(Class) Test-1: Analysis II
Statistics and Analysis (202001350)

28 September 2021, 08:45—10:15, NH-205 Total Points : 20

All answers must be motivated.
Approach to a solution is equally important as the final answer.
Use of an electronic calculator or a book is not allowed.
Good Luck!

1. (a) Give the definition of convergence of a series of real numbers. [2]

   (b) Determine the convergence/divergence of the following series. In case a series converges, determine whether it also converges absolutely. [2+2]

   (i.) $\sum_{k=1}^{\infty}(-1)^k \frac{2k-\log(k)}{k^2+\log(k)}$ (ii.) $\sum_{k=1}^{\infty}\sin(\frac{1}{k^2})\cos(\frac{1}{k})$

   (c) Prove the following assertion: [3]
   Suppose there exists an $r \in (0,1)$ such that for all $k \in \mathbb{N}$ there holds:

   $$|a_{k+2}| \leq r|a_k|.$$

   Then the series $\sum_{k=1}^{\infty} a_k$ is absolutely convergent.

2. Consider the series of real valued functions given by:

   $$\sum_{k=1}^{\infty} e^{-k^2 x}. \qquad (1)$$

   (a) Show that for every $x_0 > 0$ the series in (1) converges uniformly on $[x_0, \infty)$. [3]

   (b) Show that the series in (1) converges pointwise on $(0, \infty)$, but not uniformly on that interval. [3]

3. (a) Find the radius of convergence and the convergence interval I of the power series: [3]

   $$f(x) = \sum_{k=0}^{\infty} \frac{k!}{(2k)!} x^k.$$

   (b) Show that $f(x)$ is analytic in I. [1]

   (c) Is there a (non-empty) interval $(-a, a) \subseteq I$ such that $f(x) = 0$ for all $x \in (-a, a)$? [1]

Grade:	$\frac{\text{score on test}}{20} \times 9 + 1$ (rounded off to one decimal place)