(Class) Test-1: Analysis II
Statistics and Analysis (202001347)

27 September 2021, 08:45—10:15, Therm-1 Total Points : 20

All answers must be motivated.
Approach to a solution is equally important as the final answer.
Use of an electronic calculator or a book is not allowed.
Good Luck!

1. (a) Give the definition of absolute convergence of a series of real numbers. [2]

   (b) Determine the convergence/divergence of the following series. In case a series converges, determine whether it also converges absolutely. [2+2]

   $$\text{(i.)} \quad \sum_{k=1}^{\infty}(-1)^{2k}\frac{k^3+\log(k)}{k+\exp(k)} \qquad\qquad \text{(ii.)} \quad \sum_{k=1}^{\infty}(-1)^k[1-\exp(1/k)]$$

   (c) Prove the following assertion: [3]
   Let a_k be a positive sequence ($a_k > 0$), and suppose there exists an $r \geq 1$ such that for all $k \in \mathbb{N}$ there holds:

   $$a_{k+2} \geq r a_k.$$

   Then the series $\sum_{k=1}^{\infty} a_k$ is divergent.

2. Consider the sequence of real valued functions given by:

   $$f_n(x) = \sqrt{\sin(x/n) + x + 1}. \tag{1}$$

   (a) Show that for every $x_1 > x_0 \geq 0$ the sequence in (1) converges uniformly on $[x_0, x_1]$ and determine its limit. [3]

   (b) Show that the sequence in (1) converges pointwise on $[0, \infty)$, but not uniformly on that interval. [2]

   (c) Determine the following limit [1.5]

   $$\lim_{n\to\infty}\int_0^1 f_n(x)dx.$$

   $(x+1)^{\frac{1}{2}}$

3. (a) Find the convergence interval I of the power series: [2]

   $$f(x) = \sum_{k=0}^{\infty}\sqrt{k}x^k.$$

   k^{-1}

   $-k^{-2}$

   (b) Show that $f(x)$ is analytic in any open interval within I. [1.5]

   (c) Does $f^{(2)}(0.5)$ exist? [1]

Grade:	$\dfrac{\text{score on test}}{20} \times 9 + 1$	(rounded off to one decimal place)	