Dynamical Systems (201500103) — test 2

Date:	15-01-2018
Place:	Sports Centre, Hall 1
Time:	3 hours (plus 45 minutes for students with special rights)
Course coordinator:	Gjerrit Meinsma
Allowed aids during test:	basic calculator

1. Consider

$$\dot{x} = \begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix} x + \begin{bmatrix} \beta \\ 1 \end{bmatrix} u,$$

$$y = x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u.$$

   (a) For which $\beta \in \mathbb{R}$ is this system controllable?

   (b) For which $\beta \in \mathbb{R}$ is this system stabilizable?

   (c) Determine the reachable subspace. (The answer might depend on β.)

   (d) Take $\beta = 0$. Determine a state feedback $u = -Fx$ that places the two closed loop poles at $-1, -2$.

   (e) Take $\beta = 0$. Determine a controller with input y and output u that makes the closed loop system asymptotically stable.

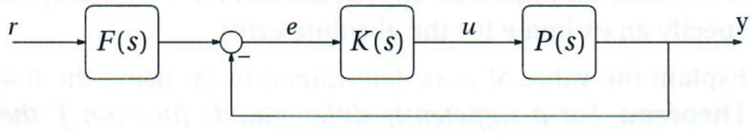
   (f) Determine the transfer matrix of the system (from u to y).

2. Write the differential equation

$$y^{(2)} - y^{(1)} = 5u^{(2)} + u$$

   in state space form $\dot{x} = Ax + Bu, y = Cx + Du$.

3. Consider the above closed loop system with $F(s), K(s)$ and $P(s)$ rational transfer functions.



   The block $F(s)$ is an asymptotically stable system.

   (a) Determine the transfer function from r to y. (Include the derivation, i.e. it is not sufficient to just give the answer.)

   (b) Let $F(s) = 1/(2s+1)$ and $P(s) = 1/(s-1)$. Determine a controller $K(s)$ that

   - has a pole at the origin
   - makes the closed loop system asymptotically stable. ($F(s)$ does not affect closed loop stability because $F(s)$ is asymptotically stable.)

   (c) Let $r(t) = \mathbb{1}(t)$. Given the $F(s), P(s)$ and $K(s)$ of the previous part, determine the steady state values $\lim_{t\to\infty} u(t)$ and $\lim_{t\to\infty} y(t)$.

4. Consider the initially-at-rest system $y = \mathcal{H}(u)$ described by

$$\dot{y}(t) = -y(t) + u(t) + u(t-1).$$

This system is LTI.

(a) Determine the maximal peak-to-peak gain $\|\mathcal{H}\|_1$ of this system.

(b) Determine the frequency response of this system.

5. Three questions.

(a) The definition of observability can also be applied to nonlinear systems. Is the nonlinear system

$$\dot{x} = x + u, \qquad y = x^3 + u$$

observable?

(b) What is the definition of *detectability*?

(c) Is the system $y(t) = \sin(t)u(t)$ time invariant?

6. **Numerical Methods**: Using the trapezoidal rule for integration of the function

$$f(x) = 2\,\mathrm{e}^{-x-x^2} + (3\,\mathrm{e}^{-2} - 1)x^2$$

on the interval $[0,1]$, i.e., $I = \int_0^1 f(x)\,\mathrm{d}x$, we obtain numerical approximations $I(h)$ at step size h as given in the following table:

h	numerical value $I(h)$
0.5000	0.817286388000510
0.2500	0.816212596855330
0.1250	0.816148425997833
0.0625	0.816144458517675

(a) Determine from these data the order of convergence of this proces, i.e., determine the value of p in the relation $I(h) = I + a_p h^p + O(h^{p+1})$.

(b) Determine an improved approximation for I by extrapolating once. Also specify an estimate for the absolute error.

(c) Explain the value of p as determined in (a) using the following theorem.
Theorem: *For a sufficiently differentiable function f the approximation $I(h)$ for the integral as obtained by the trapezoidal rule obeys*

$$I(h) = \int_0^1 f(x)\,\mathrm{d}x + a_2h^2 + a_4h^4 + \cdots + a_{2m}h^{2m} + O(h^{2m+2}),$$

where $a_{2k} = \dfrac{b_{2k}}{(2k)!}(f^{(2k-1)}(1) - f^{(2k-1)}(0))$ *and known numbers* b_{2k}.

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Tentamencijfer: $1 + 9p/36$.