Dynamical Systems (201500103) — test 2

Date:	14-01-2019
Time:	8:45–11:45 (45 minutes extra for students with special rights)
Place:	Therm
Course coordinator:	Gjerrit Meinsma
Allowed aids during test:	a basic calculator

1. Consider the system described by the differential equation

$$\ddot{y} - 2\dot{y} + y = \dot{u} + \beta u. \tag{1}$$

   (a) Determine an equivalent state model

$$\begin{aligned} \dot{x} &= Ax + Bu, \\ y &= Cx + Du. \end{aligned} \tag{2}$$

   (Equivalent means: (u, y) satisfies (1) if-and-only-if there is an x such that (u, x, y) satisfies (2).)

   (b) For which β is state model (2) controllable?

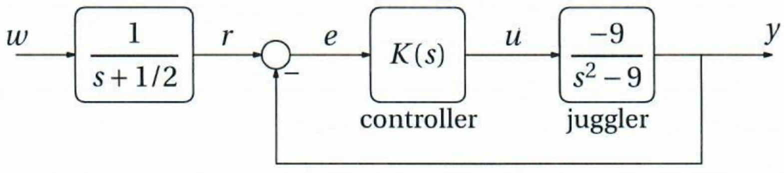
   (c) Determine the reachable subspace of state model (2).
   (The answer might depend on β)

   (d) For which β is state model (2) observable?

   (e) Determine an observer, with observer poles -1 and -2, for state model (2).

   (f) Take $\beta = 0$. Determine a controller with input y and output u that makes the closed loop system asymptotically stable.

2. Consider the familiar "juggler"control problem, including a prefilter $\dfrac{1}{s+1/2}$:



   In what follows assume that the closed loop is asymptotically stable, that all signals are zero for $t < 0$, and that $w(t) = \mathbb{1}(t)$. Recall that its Laplace transform then equals $W(s) = \int_0^\infty \mathbb{1}(t)\,\mathrm{e}^{-st}\,\mathrm{d}t = 1/s$.

   (a) Determine $r_\infty := \lim_{t\to\infty} r(t)$.

   (b) Show that the Laplace transform $E(s)$ of $e := r - y$ has the property that $E(3) = 0$.

   (c) It is often desirable to try to keep $y(t) \le r(t)$ for all time. Is there a stabilizing controller $K(s)$ that achieves this for our juggler problem?

3. Consider the system

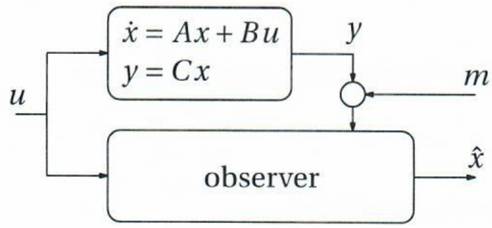
$$y(t) = \int_{-1}^{1} \tau u(t-\tau)\,\mathrm{d}\tau.$$

This system is LTI.

(a) Determine the maximal peak-to-peak gain $\|\mathcal{H}\|_1$.

(b) Determine the frequency response of this system.

4.



The above is the familiar observer block diagram, except for the presence of "measurement noise" m.

Suppose $\dot{x} = Ax + Bu$ is asymptotically stable. Is there an observer with the property that $\lim_{t\to\infty} x(t) - \hat{x}(t) = 0$ for every $x(0)$, u and m?

5. Three questions.

(a) Formulate the *Kalman Observability Decomposition* theorem.

(b) Is the system $y(t) = u(-t)$ linear?

(c) Is the system $y(t) = u(-t)$ time-invariant?

6. **Numerical Methods**: Consider the function $f : \mathbb{R} \to \mathbb{R}$

$$f(x) = \cos(x) + \mathrm{e}^{-x} \qquad (3)$$

(a) Prove that f has at least one root on the interval $[0, \pi]$.

We compute a root of f by iteration.

(b) i. Which numerical method would you select to be guaranteed to find one root on the interval $[0, \pi]$?

ii. Estimate the number of steps needed with this method to approximate a root with an accuracy of 10^{-4}.

(c) An efficient method to determine roots is Newton's method.

i. Starting from $x = 0$, compute the result of applying Newton's method once for the function f in (3)?

ii. Argue how many steps are needed to approximate a root with an accuracy of 10^{-4} using Newton's method.

opgave:	1	2	3	4	5	6
punten:	2+2+2+2+2+2	2+2+2	2+2	2	2+2+2	6

Exam grade: $1 + 9p/36$.