Dynamical Systems (201500103) — test 2 (resit)

Date:	01-02-2019
Time:	8:45–11:45 (45 minutes extra for students with special rights)
Place:	CI H327
Course coordinator:	Gjerrit Meinsma
Allowed aids during test:	a basic calculator

1. Consider the system described by the differential equation

$$\ddot{y} - 2\dot{y} - 3y = \beta\dot{u} + u. \tag{1}$$

   (a) Determine an equivalent state model

$$\begin{aligned} \dot{x} &= Ax + Bu, \\ y &= Cx + Du. \end{aligned} \tag{2}$$

   (Equivalent means: (u, y) satisfies (1) if-and-only-if there is an x such that (u, x, y) satisfies (2).)

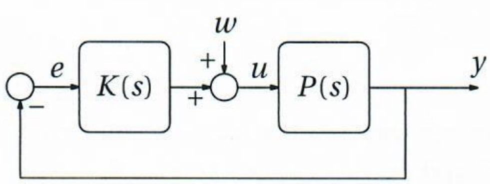
   (b) For which β is state model (2) controllable?

   (c) For which β is state model (2) stabilizable?

   (d) Let $\beta = 1$. Determine an F such that $A - BF$ has eigenvalues -1 (twice).

   (e) Take $\beta = 1$. Determine a controller with input y and output u that makes the closed loop system asymptotically stable.

2. Consider the system with disturbance w:



   (a) Determine the transfer function from w to u.

   (b) Suppose $P(s) = 1, K(s) = 1/s$, and that $w(t) = \sin(t)$. Determine numbers A, ω, b such that $\lim_{t\to\infty} y(t) - A\sin(\omega t + b) = 0$.

   (c) Suppose $P(s) = 1$ and $w(t) = \sin(t)$, and that $K(s)$ stabilizes the closed loop system and such that $\lim_{t\to\infty} y(t) = 0$. Show that $K(s)$ must have the form

$$K(s) = \frac{N_K(s)}{(s^2+1)D_K(s)}$$

   for certain polynomials $N_K(s), D_K(s)$.

   (d) Suppose $P(s) = 1$, and that $w(t) = \sin(t)$. Determine a stabilizing controller $K(s)$ such that $\lim_{t\to\infty} y(t) = 0$.

3. Consider

$$\dot{x} = \begin{bmatrix} 3 & -4 \\ 2 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & -2 \end{bmatrix} x$$

(a) Determine the Kalman Observability Decomposition of this system.

(b) Determine the impulse response of this system (assumed initially-at-rest). 4

4. Let $A \in \mathbb{R}^{n\times n}$ and $C \in \mathbb{R}^{k\times n}$. Consider the **discrete time** system

$$x(t+1) = Ax(t), \qquad t \in \mathbb{Z},$$
$$y(t) = Cx(t).$$

This discrete-time system is said to be *observable* if there is a $T \in \mathbb{N}$ such that $x(0)$ follows uniquely from $y(0), y(1), \ldots, y(T)$.

Prove that the discrete-time system is observable if-and-only-if the *observability matrix* (as we know from our course) has rank n.

5. Three questions.

(a) What is the *frequency response* of an initially-at-rest system described by $\dot{x} = Ax + Bu, y = Cx + Du$?

(b) Is the system $y(t) = \mathrm{e}^t u(t)$ linear?

(c) Is the system $y(t) = \mathrm{e}^t u(t)$ time-invariant?

6. **Numerical Methods**: Consider the problem

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(t, y), \qquad y(0) = y_0,$$

for a given smooth function f and a number y_0. The task is to compute $y(1)$.

(a) Determine the analytical solution in case

$$f(t, y) = -3y, \qquad y_0 = 1. \tag{3}$$

The midpoint rule could be used to approximate $y(1)$. Denoting with y_n the numerical approximation at time $t_n = nh$, with h the step size, the midpoint rule can be expressed as:

$$y_{n+1} = y_n + hf(t_{n+1/2}, \frac{1}{2}(y_n + y_{n+1})).$$

(b) Is this method explicit or implicit?

(c) Show that the local truncation error of the method is of order 3 for the problem as described in (3).

(d) If the local truncation error of the method is of order 3 for any smooth f, what could you conclude for the global truncation error?

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Exam grade: $1 + 9p/36$.