Dynamical Systems (201500103) — test 2

Date:	13-01-2020
Time:	8:45–11:45 (45 minutes extra for students with special rights)
Place:	SP 4
Course coordinator:	Gjerrit Meinsma
Allowed aids during test:	a basic calculator

1. Consider the system described by

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix} x + \begin{bmatrix} 1 \\ \beta \end{bmatrix} u \\ y &= \begin{bmatrix} 0 & 1 \end{bmatrix} x. \end{aligned} \tag{1}$$

   (a) For which β is state model (1) controllable?

   (b) Determine the reachable subspace of state model (1). (The answer might depend on β)

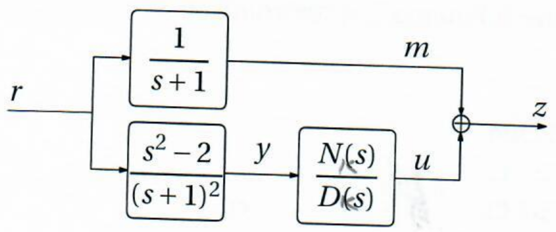
   (c) For which β is state model (1) observable?

   (d) Determine for (1) an observer, with observer poles -1 and -3.

   (e) Take $\beta = -2$. Determine a controller with input y and output u that makes the closed loop system asymptotically stable.

   (f) Determine a differential equation $\ddot{y} + p_1\dot{y} + p_0 y = q_1\dot{u} + q_0 u$ whose observer canonical form equals (1).

2. Consider the "filtering" configuration



Here $z = m + u$, and $N(s)$ and $D(s)$ are polynomials.

   (a) Determine $H_{z/r}(s)$

   (b) This system has input r and output m, y, u. Prove that the above interconnected system is asymptotically stable if-and-only-if $D_K(s)$ is an asymptotically stable polynomial.

   (c) Determine a constant $K(s)$ such that the mapping from r to z has zero DC-gain.

   (d) Let $r(t) = \mathbb{1}(t)\cos(\omega t)$. For which $\omega > 0$ is there no asymptotically stable $K(s)$ such that $\lim_{t\to\infty} z(t) = 0$?

3. Let $u, y : \mathbb{R} \to \mathbb{R}$. Consider the system $y = \mathscr{H}(u)$ defined by

$$y(t) = \int_{-1}^{1} \tau^2 u(t-\tau)\,\mathrm{d}\tau.$$

   (a) Is this system linear?

   (b) Is this system time invariant?

   (c) Determine the maximal peak-to-peak gain $\|\mathscr{H}\|_1$.

4. Three questions.

   (a) Formulate the *Kalman Observability Decomposition* theorem.

   (b) Give a system that is detectable but not observable.

   (c) Suppose a 5×5 matrix A has 5 different eigenvalues, of which 2 have positive real part. If $\dot{x} = Ax + Bu$ is stabilizable, then what you say about the rank of the controllability matrix?

5. **Numerical Methods 1**:

   (a) What is the expression for the condition number of the problem: "compute the value of the function f in a point x"? Compute the condition number in case f is given by

$$f(x) = \tanh(x) = \frac{\mathrm{e}^x - \mathrm{e}^{-x}}{\mathrm{e}^x + \mathrm{e}^{-x}}.$$

   (b) Consider $x = 0.68$ with a possible absolute error of 0.01. We wish to compute $f(x)$.

      i. What is the value of the condition number in this case?

      ii. Given the condition number from 5(b)i, what can you conclude regarding the relative error with which $f(0.68)$ is computed?

6. **Numerical Methods 2**: with the help of a numerical integration process we obtain for a certain integral I the following approximations $I(h)$ as function of step size h:

h	$I(h)$
1/2	3.26914555200204
1/4	3.26485038742132
1/8	3.26459370399133
1/16	3.26457783407070

   (a) Determine the order p of the numerical integration process based on the data in the table, i.e., determine the value of p from the expression $I(h) = I + c\,h^p + O(h^{p+1})$, $p \in \mathbb{N}$.

   (b) Perform one extrapolation to obtain an improved approximation for I. Include an estimate of the absolute error.

problem:	1	2	3	4	5	6
points:	2+2+2+2+2+2	1+2+1+2	2+2+2	2+2+2	1.5+1.5	1+2

Exam grade: $1 + 9p/36$.