Dynamical Systems (201500103) — Exam02-RESIT

Date:	31-01-2020
Time:	8:45–11:45 (45 minutes extra for students with special rights)
Place:	Sports Centre SC2
Course coordinator:	Gjerrit Meinsma
Allowed aids during test:	a basic calculator

1. Let $\alpha \in \mathbb{R}$. Consider

$$\dot{x} = \begin{bmatrix} 4 & -6 \\ \alpha & -5 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u, \qquad (1)$$
$$y = \begin{bmatrix} -1 & 1 \end{bmatrix} x.$$

   (a) For which α's is the system controllable?

   (b) For which α's is the system observable?

   (c) Is het systeem detectable if $\alpha = 3$?

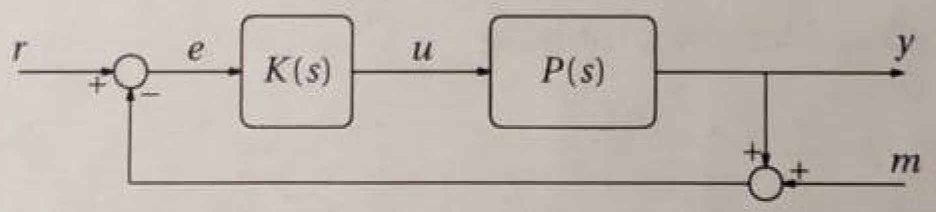
   (d) Take $\alpha = 0$. Determine an observer for this system.

   (e) Take $\alpha = 0$. Determine a controller that stabilizes the closed loop system.

   (f) Determine the transfer function of this system.

   (g) Take $\alpha = 0$. It can be shown that (1) is equivalent to a differential equation $\ddot{y} + p_1 \dot{y} + p_0 y = q_1 \dot{u} + q_0 u$. Determine the values of p_1, p_0, q_1, q_0. 8

2. Consider the following closed loop system in which $K(s)$ and $P(s)$ are rational transfer functions, $K(s) = \frac{N_K(s)}{D_K(s)}$, $P(s) = \frac{N_P(s)}{D_P(s)}$:



The signal m models a measurement error (the difference between the actual output y and the measurement of this output). 5

   (a) Determine the transfer function from m to y.

   (b) Let $P(s) = 1/s^2$. Determine a controller $K(s)$ that stabilizes the closed loop.

   (c) Let $P(s) = 1/s^2$ and suppose the closed loop system is asymptotically stable. Let $r(t) = 0$ and $m(t) = m_0$ (a constant). Determine $\lim_{t\to\infty} y(t)$.

3. Consider the initially-at-rest system

$$\dot{y}(t) = -y(t) + \int_{t-2}^{t} u(\tau)\,\mathrm{d}\tau.$$

This system is LTI.

   (a) Show that the system is BIBO stable.

   (b) Determine the frequency response $H_{y/u}(\mathrm{i}\omega)$.

   (c) Determine the maximal peak-to-peak gain $\|\mathscr{H}\|_1$.

4. Three questions.

   (a) Give the definition of *controllability*.

   (b) Is the system $y(t) = u(-t^2)$ LTI?

   (c) Is this *nonlinear* system 5

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix} x, \qquad y = e^{x_2}$$

   detectable?

5. **Numerical Mathematics:** Consider $f : \mathbb{R} \to \mathbb{R}$

$$f(x) = \sin(x) - 2\cos(x^2) + 1. \tag{2}$$

   (a) Sketch f on the interval $[0, 1]$ and prove that f has at least one root on this interval.

   We compute a root of f by iteration.

   (b) i. Which numerical method would you select to be guaranteed to find one root on the interval $[0, 1]$?

   ii. Estimate the number of steps needed with this method to approximate a root with an accuracy of 10^{-4}. 3

   (c) An efficient method to determine roots is Newton's method.

   i. Starting from $x = 0$, compute the result of applying Newton's method once for the function f in (2)?

   ii. How many steps are needed to approximate a root with an accuracy of 10^{-4} using Newton's method?

6. **Numerical Mathematics:** Consider the problem

$$\frac{dy}{dt} = f(t, y), \quad y(0) = y_0,$$

   with a given smooth function f and y_0 an arbitrary number. The midpoint rule may be used to solve this problem. Writing the numerical solution at $t_n = nh$ with h the step size as y_n the method proceeds as follows:

$$y_{n+1} = y_n + hf(t_{n+1/2}, \frac{1}{2}(y_n + y_{n+1})),$$

   with $t_{n+1/2} = (t_n + t_{n+1})/2$.

   (a) Is this method explicit or implicit?

   (b) Assume $f(t, y) = -y$. Show that the local truncation error is of third order.

   (c) What is the order of the global error?

   (d) How can Newton iteration be used to iteratively determine y_{n+1}? What starting value would you suggest for the iteration and what stopping criterion?

problem:	1	2	3	4	5	6
points:	2+2+2+2+2+2+2	1+2+2	2+2+2	2+2+1	3	3

Exam grade: $1 + 9p/36$.