

AM-M6-NM Test 1: Numerical Methods

Course : AM-M6 - Numerical Mathematics (202001356)
Module : Dynamical Systems
Date : Friday December 4, 2020
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Time : 09:00 - 09:30 (09:38) uur
Duration : 30 min (in case of extra time: 38 min)

Notice:

- Motivate your answers.
- This test consists of 2 pages, including this one, and contains 2 exercises.
- For this test you can get a grade $= 1 + \# \text{points}$ with maximally 9 points distributed over the exercise as detailed below.
- Only use UT exam paper. Write your name and student number on each sheet of paper. Do not hand in your notes on scratch paper.

Points rewarded:

Exercise	Points
1a	2
1b	1
1c	1
2a	2
2b	3

Grade $= 1 + \# \text{points}$

Exercises Numerical Mathematics

Exercise 1.

- (a) What is the expression for the condition number of the problem: ‘compute the value of the function f in a point x ’? Compute the condition number in case f is given by

$$f(x) = \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

- (b) Consider $x = 0.68$ with a possible absolute error of 0.01. We wish to compute $f(x)$. What is the value of the condition number in this case?
- (c) Given the condition number from (b), what can you conclude regarding the relative error with which $f(0.68)$ is computed?

Exercise 2. We want to approximate a quantity $I(0)$ and obtain a sequence of numerical estimates $I(h)$ at step sizes h as given in the following table:

h	numerical value $I(h)$
0.5000	0.817286388000510
0.2500	0.816212596855330
0.1250	0.816148425997833
0.0625	0.816144458517675

- (a) Determine from these values the order of convergence of this proces, i.e., determine the value of p in the relation

$$I(h) = I(0) + ah^p + O(h^{p+1}).$$

- (b) Determine the best approximation for $I(0)$ from this information by extrapolating once. Also specify an estimate for the absolute error.