Systems Theory (202001355)

Date:	11-01-2021
Time:	9:45–12:15 (till 12:52 for students with special rights)
Place:	RA 2502 & RA 2504
Course coordinator:	Gjerrit Meinsma
Allowed aids during test:	a basic calculator

1. Consider the system described by

$$\dot{x} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ \beta \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x. \qquad (1)$$

   (a) For which β is system stabilizable?

   (b) Determine the reachable subspace of this system. (The answer might depend on β)

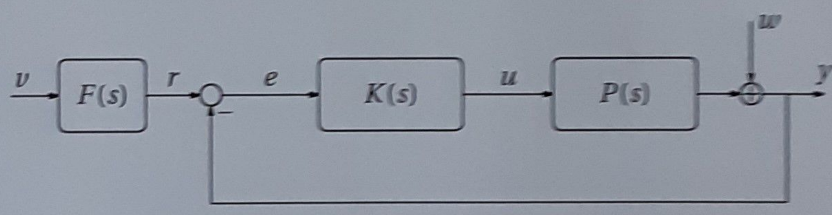
   (c) Is the system observable?

   (d) Suppose β is such that the system is controllable. Determine a stabilizing state feedback $u = -Fx$.

   (e) For this system determine an observer with both observer poles equal to -1.

   (f) Determine the transfer function from u to y.

2. Consider the following configuration



   (a) Determine $H_{y/v}(s)$.

   (b) Now let $F(s) = 1$, in other words, we are back at the standard error feedback scheme (and $v = r$). Suppose

$$K(s) = \frac{as+b}{s}, \qquad P(s) = \frac{1-s}{1+s}.$$

   Determine $a, b \in \mathbb{R}$ such that the closed loop characteristic polynomial equals $c(s+1)^2$ for some nonzero constant c.

   (c) Let $F(s) = 1$ and let $K(s)$ be as in the part (b) of this problem. Suppose $r = \mathbb{1}(t)$ and $w = 4\mathbb{1}(t)$. Determine $\lim_{t\to\infty} y(t)$.

3. Let $u, y : \mathbb{R} \to \mathbb{R}$. Consider the system $y = \mathcal{H}(u)$ defined by

$$y(t) = \int_{-1}^{1} u(\tau - t)\,\mathrm{d}\tau.$$

(a) Is this system linear?

(b) Is this system time invariant?

(c) Determine the maximal peak-to-peak gain $\|\mathcal{H}\|_1$.

4. Four questions.

(a) Give the definition of *observability* of a system $\dot{x} = Ax + Bu, y = Cx + Du$.

(b) Is the *nonlinear* system $\dot{x}(t) = x^2(t)u(t) + u^3(t)$ controllable?

(c) Determine the Kalman controllability decomposition of

$$\dot{x} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u,$$
$$y = \begin{bmatrix} 3 & 4 \end{bmatrix} x.$$

(d) Determine an equivalent state representation of

$$\dddot{y} - 2\dot{y} = \ddot{u} + 3\dot{u} - 2u.$$

problem:	1	2	3	4
points:	2+2+2+2+2+1	2+2+2	1+2+2	2+2+2+2

Exam grade: $1 + 9p/30$.