AM-M6-NM Test 2: Numerical Methods

Course	: AM-M6 - Numerical Mathematics (202001356)
Module	: Dynamical Systems
Date	: Monday January 11, 2021
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Time	: 09:00 - 09:30 (09:38) uur
Duration	: 30 min (in case of extra time: 38 min)

Notice:

- Motivate your answers.
- This test consists of 2 pages, including this one, and contains 2 composite questions.
- For this test you can get a grade = 1+#points with maximally 9 points distributed over the various sub-questions as detailed below.
- Please, use only UT exam paper. Write your name and student number on each sheet of paper. Do not hand in your notes on scratch paper.
- You are allowed to use your graphical calculator.

Points rewarded:

Exercise	Points
1a	2
1b	2
1c	3
2a	1
2b	1

Grade = 1+#points

Numerical Mathematics

Question 1.

Consider $f : \mathbb{R} \to \mathbb{R}$

$$f(x) = \exp(-x^2) - x$$

a) Sketch f and df/dx on the interval $[0, 1]$ and prove that f has exactly one positive root.

Hint: use the mean value theorem and the fact that $1/e < 1$.

We compute the root of f by iteration.

b) Starting from the interval $[0, 1]$, how many steps will the bisection method require to approximate the root with an accuracy of 10^{-4}.

c) Formulate Newton's method to determine the root of f and compute the first iteration starting from $x_0 = 1$.

Question 2.

Consider the problem

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(t, y), \quad y(0) = y_0,$$

with a given smooth function f and y_0 an arbitrary number. The midpoint rule may be used to solve this problem. Writing the numerical solution at $t_n = nh$ with h the step size as y_n the method proceeds as follows:

$$y_{n+1} = y_n + hf(t_{n+1/2}, \frac{1}{2}(y_n + y_{n+1})).$$

with $t_{n+1/2} = (t_n + t_{n+1})/2$

(a) Is this method explicit or implicit?

(b) The local truncation error is of third order. What is the order of the global error?