Systems Theory (202001355)

Date:	29-01-2021
Time:	9:45–12:15 (till 12:52 for students with special rights)
Place:	RA 2334
Course coordinator:	Gjerrit Meinsma
Allowed aids during test:	a basic calculator

1. Let $\alpha, \beta \in \mathbb{R}$. Consider the system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} x. \end{aligned} \tag{1}$$

   Notice that y is a vector with *two* entries.

   (a) For which α, β is system controllable?

   (b) For which α, β is system stabilizable?

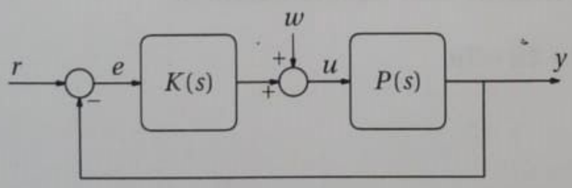
   (c) For which α, β is the system observable?

   (d) Let $\alpha = \beta = -1$ determine *all* state feedback laws $u = -Fx$ that place the two poles at -1 and -2.

   (e) Let $\alpha = \beta = -1$. Determine an observer with both observer poles -2.

   (f) Determine the transfer matrix from u to y.

2. Consider the following configuration



   (a) Determine the transfer function from w to y.

   (b) Let $P(s) = s^2/(s^2+1)$ and $w = 0$. Are there stabilizing controllers $K(s)$ such that $\lim_{t\to\infty} y(t) = r_0$ if $r(t) = r_0 \mathbb{1}(t)$?

   (c) Recall that a polynomial $Q(s) := q_3 s^3 + q_2 s^2 + q_1 s + q_0$ is asymptotically stable iff q_3, q_2, q_1, q_0 have the same sign and $q_2 q_1 > q_3 q_0$. Let $P(s) = s^2/(s^2+1)$. Determine all $K(s)$ of the form $K(s) = 1/(as+b)$ that stabilize the closed loop.

3. Let $u, y: \mathbb{R} \to \mathbb{R}$. Consider the system $y = \mathscr{H}(u)$ defined by

$$y(t) = u(\alpha t + \beta).$$

Here, α, β are real numbers.

(a) For which α, β is the system linear?

(b) For which α, β is the system time-invariant?

(c) "Causality" roughly speaking means that for each $t_0 \in \mathbb{R}$ the output $y(t_0)$ is a function of "the past" $\{u(t) | t \leq t_0\}$ of the input. For which α, β is our system causal?

4. Four questions.

(a) Formulate the *Hautus test* for observability of a system $\dot{x} = Ax + Bu, y = Cx + Du$.

(b) Suppose a system $y = \mathscr{H}(u)$ is LTI and BIBO-stable, and let h be its impulse response. Show that the response $y(t)$ to $u(t) = \mathrm{e}^{\mathrm{i}\omega t}\mathbb{1}(t)$ exists and equals

$$y(t) = \left(\int_{-\infty}^{t} h(\tau)\,\mathrm{e}^{-\mathrm{i}\omega\tau}\,\mathrm{d}\tau \right) \mathrm{e}^{\mathrm{i}\omega t}.$$

In your derivation indicate where you use LTI and/or BIBO-stability.

(c) Determine the Kalman observability decomposition of

$$\dot{x} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} x + \begin{bmatrix} 3 \\ 4 \end{bmatrix} u,$$
$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} x.$$

(d) Determine an equivalent state representation of

$$\dddot{y} - 2\dot{y} = 2\ddot{u} + 2\dot{u} - 3u.$$

problem:	1	2	3	4
points:	2+2+2+2+2+1	2+2+2	2+2+2	1+2+2+2

Exam grade: $1 + 9p/30$.