# AM-M6 Test 2 Resit: Numerical Methods

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| Course | : AM-M6 - Numerical Mathematics (202001356) |
| Module | : Dynamical Systems |
| Date | : Friday January 29, 2021 |
| Author | : Bernard Geurts |
| Time | : 09:00 - 09:30 (09:38) uur |
| Duration | : 30 min (in case of extra time: 38 min) |

**Notice:**

- Motivate your answers.
- This test consists of 2 pages, including this one, and contains 2 composite questions.
- For this test you can get a grade = 1+#points with maximally 9 points distributed over the various sub-questions as detailed below.
- Please, use only UT exam paper. Write your name and student number on each sheet of paper. Do not hand in your notes on scratch paper.
- You are allowed to use your graphical calculator.

**Points rewarded:**

| Exercise | Points |
|---|---|
| 1a | 3 |
| 1b | 1 |
| 1c | 2 |
| 2a | 1 |
| 2b | 2 |

Grade = 1+#points

## Numerical Mathematics

**Question 1.**

Consider the integral $I$ of a twice differentiable function $f : \mathbb{R} \to \mathbb{R}$ over the interval $[a, b]$

$$I = \int_a^b f(x)\,dx$$

The integral will be approximated using the midpoint rule $I_m$ in which

$$I \approx I_m := (b - a)f(c)$$

where $c = (a + b)/2$.

a) Determine the quadrature error $E = I - I_m$.
   Hint: use the Taylor expansion of $f$ around $c$ to represent the integrand $f$.

b) For which functions $f$ is the mid-point rule exact?

c) Divide the interval $[a, b]$ into intervals $[x_i, x_{i+1}]$ for $i = 0, ..., n - 1$ where $x_0 = a$, $x_i < x_{i+1}$ and $x_n = b$. Formulate the composite integration method corresponding to the midpoint rule applied to this division of $[a, b]$.

**Question 2.**

Consider the Cauchy problem

$$\frac{dy}{dt} = f(y(t), t) \quad \text{for } t > t_0 \quad \text{with } y(t_0) = y_0$$

Here, $f$ is a continuous, real-valued function of both its variables. The solution $y(t_n)$ to this problem at time $t_n = nh$, where $h$ denotes the step size, will be approximated by $u_n \approx y(t_n)$ using the trapezoidal rule in which

$$u_{n+1} = u_n + \frac{h}{2}\Big(f_n + f_{n+1}\Big)$$

where $f_n = f(u_n, t_n)$ for $n = 0, 1, ...$ .

(a) Is the trapezoidal rule explicit or implicit?

(b) Apply the trapezoidal rule to the problem in which $f(y(t), t) = \lambda y(t)$ where $\lambda \in \mathbb{R}$. The numerical solution is called absolutely stable if

$$\lim_{n\to\infty} |u_n| = 0$$

For which $\lambda h$ is the trapezoidal rule absolutely stable?