Systems Theory (202001355)

Date:	17-01-2022
Time:	8:45–11:45 (till 12:30 for students with special rights)
Place:	OH 211
Course coordinator:	Gjerrit Meinsma
Allowed aids during test:	a basic calculator

1. Let $\alpha, \beta \in \mathbb{R}$ and consider the system with two output components described by

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t) \\ y(t) &= x(t). \end{aligned} \tag{1}$$

   (a) For which α, β is system stabilizable?

   (b) For which α, β is the system observable?

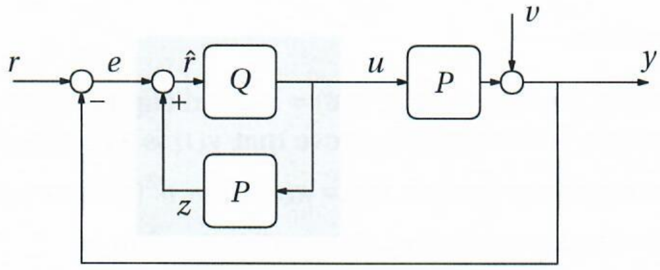
   (c) Let $\alpha = 1, \beta = -1$. Determine a stabilizing state feedback $u(t) = -Fx(t)$ that places the eigenvalues of the closed loop system at -2 (double).

   (d) Let α, β be such that the system is observable. Determine an observer with both observer poles poles equal to -1.

   (e) Determine the transfer function from u to y_1 (here y_1 is the first component of y).

2. What is the definition of *detectability*?

3. "Know thy enemy" is a saying in English which, when applied to control theory, means that for succesful control of a given system, the controler must contain a copy of the given system. One popular such form is this closed loop system:



   The gray box is the controller, and it contains a copy of the given system P. Notice that here $\hat{r} = e + z$.

   (a) Show that $H_{y/r}(s) = P(s)Q(s)$.

   (b) Why is it usually desirable to have $Q(0) = 1/P(0)$?

From now on suppose that

$$P(s) = \frac{1}{s+2}, \qquad Q(s) = \frac{q_0}{s+1}.$$

(c) Express the controller transfer function $K_{u/e}$ (i.e. the grey box with input e and output u) as a fraction of polynomials

$$K_{u/e}(s) = \frac{N_K(s)}{D_K(s)},$$

and argue that this controller stabilizes the closed loop system for every $q_0 \in \mathbb{R}$.

(d) Suppose $v(t) = \mathbb{1}(t)$ and $r(t) = 0$. For which q_0 do we have $\lim_{t\to\infty} y(t) = 0$?

4. Let $u, y : \mathbb{R} \to \mathbb{R}$. Consider the system $y = \mathscr{H}(u)$ defined by

$$y(t) = \int_{t-2}^{t+2} u(\tau - t)\,\mathrm{d}\tau.$$

(a) Is this system linear?

(b) Is this system time invariant?

(c) Determine the maximal peak-to-peak gain $\|\mathscr{H}\|_1$.

5. *LQ control.* Consider the scalar system

$$\dot{x}(t) = 3x(t) + 2u(t), \quad x(0) = x_0.$$

Determine the input u (as a function of x) that minimizes

$$\int_0^\infty 4x^2(t) + u^2(t)\,\mathrm{d}t$$

over all stabilizing inputs.

[Hint: the Algebraic Riccati Equation is $A^{\mathrm{T}}P + PA + C^{\mathrm{T}}C - PBB^{\mathrm{T}}P = 0$.]

6. Three questions.

(a) Consider $\dot{x} = Ax,\ y = Cx,\ x(0) = x_0$, and suppose it is observable and that the matrix A is invertible. Prove that $y(t)$ is a constant solution iff $x_0 = 0$.

(b) Is the *nonlinear* system $\dot{x}(t) = x(t)u(t) + u^2(t)$ controllable?

(c) Determine an equivalent state representation of

$$\dddot{y}(t) - 3\dot{y}(t) + 2y(t) = \ddot{u}(t) + 5u(t).$$

problem:	1	2	3	4	5	6
points:	2+2+2+2+2	2	2+2+2+2	2+2+2	3	3+2+2

Exam grade: $1 + 9p/36$.