AM-M6-NUM: Numerical Mathematics

Course	: AM-M6 - Numerical Mathematics (202001356)
Module	: Dynamical Systems
Date	: Thursday, January 20, 2022
Author	: Bernard Geurts
Time	: 08:45 - 10:45 (11:15)
Duration	: 120 min (in case of extra time: 150 min)

Notice:

- Always motivate your answers.
- This test consists of 2 pages, including this one, and contains 3 exercises.
- For this test you can get a grade = 1+#points with maximally 9 points distributed over the exercise as detailed below.
- Use only UT exam paper. Write your name and student number on each sheet of paper. Do not hand in your notes on scratch paper.

Points rewarded:

Exercise	Points
1a	1.5
1b	0.5
1c	1
2a	2
2b	1
3a	1.5
3b	0.5
3c	1

Grade = 1+#points

Exam Questions Numerical Mathematics

Exercise 1.

(a) What is the expression for the condition number of the problem: 'compute the value of the function f in a point x'? Compute the condition number in case f is given by

$$f(x) = \sinh(2x) = \frac{1}{2}(e^{2x} - e^{-2x})$$

(b) Consider $x = \sqrt{2}/2$ with a possible absolute error of 0.01. We wish to compute $f(x)$. What is the value of the condition number in this case?

(c) Given the condition number from (b), what can you conclude regarding the relative error with which $f(\sqrt{2}/2)$ is computed?

Exercise 2. We want to approximate a quantity $I(0)$ and obtain a sequence of numerical estimates $I(h)$ at step sizes h as given in the following table:

h	numerical value $I(h)$
0.5000	0.817286388000510
0.2500	0.816212596855330
0.1250	0.816148425997833
0.0625	0.816144458517675

(a) Determine from these values the order of convergence of this proces, i.e., determine the value of p in the relation

$$I(h) = I(0) + ah^p + O(h^{p+1}).$$

(b) Determine the best approximation for $I(0)$ from this information by extrapolating once. Also, specify an estimate for the absolute error.

Exercise 3. Consider the integral I of a twice differentiable function $f : \mathbb{R} \to \mathbb{R}$ over the interval $[a, b]$

$$I = \int_a^b f(x)\, dx$$

The integral will be approximated using the midpoint rule I_m in which

$$I \approx I_m := (b - a)f(c)$$

where $c = (a + b)/2$.

a) Determine the quadrature error $E = I - I_m$.
Hint: use the Taylor expansion of f around c to represent the integrand f.

b) For which functions f is the mid-point rule exact?

c) Divide the interval $[a, b]$ into intervals $[x_i, x_{i+1}]$ for $i = 0, ..., n - 1$ where $x_0 = a$, $x_i < x_{i+1}$ and $x_n = b$. Formulate the composite integration method corresponding to the midpoint rule applied to this division of $[a, b]$.