Systems Theory (202001355) — resit

Date:	04-02-2022
Time:	8:45–11:45 (till 12:30 for students with special rights)
Place:	OH 211
Course coordinator:	Gjerrit Meinsma
Allowed aids during test:	a basic calculator

1. Let $b_1, b_2, c_1, c_2 \in \mathbb{R}$ and consider the system described by

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} c_1 & c_2 \end{bmatrix} x(t). \end{aligned} \tag{1}$$

   (a) Determine the transfer function from u to y.

   (b) For which b_1, b_2 is system controllable?

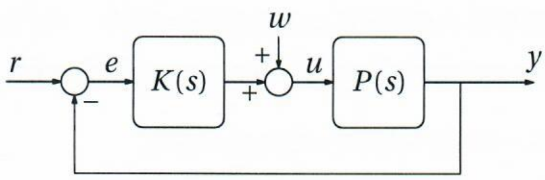
   (c) For which c_1, c_2 is the system detectable?

   (d) Now take $b_1 = 0, b_2 = 1$. Determine a stabilizing state feedback $u(t) = -Fx(t)$ that places the two eigenvalues of the closed loop system at -1 and -2.

   (e) Now take $b_1 = 0, b_2 = 1, c_1 = 0, c_2 = 1$. Determine a stabilizing controller formed from an observer and state feedback.

2. What is the definition of the *maximal peak-to-peak gain* of a linear system $y = \mathscr{H}(u)$? You may assume that $u, y : \mathbb{R} \to \mathbb{R}$.

3. Consider the system



   (a) Determine the transfer function from w to y.

   (b) Suppose $K = 1/(s+1)$ stabilizes the closed loop and that $r(t) = 0$ and $w(t) = \mathbb{1}(t)$. Show that $\lim_{t\to\infty} y(t) = 0$ if-and-only-if $P(0) = 0$.

   (c) Suppose $P(s) = s/(s+1)$ and that $r(t) = r_0, w(t) = 0$. Is there a stabilizing controller $K(s)$ such that $\lim_{t\to\infty} y(t) = r_0$, irrespective of the choice of r_0?

4. Let $u, y : \mathbb{R} \to \mathbb{R}$. Consider the system $y = \mathscr{H}(u)$ defined by

$$y(t) = \int_{t-3}^{t-1} u^2(\tau)\,\mathrm{d}\tau.$$

   (a) Is this system linear?

   (b) Is this system time invariant?

5. *LQ control.* Let $c \in \mathbb{R}$. Consider the scalar system

$$\dot{x}(t) = x(t) + u(t), \quad y(t) = cx(t), \quad x(0) = x_0.$$

   (a) Suppose $c \neq 0$. Determine the input u (as a function of x) that minimizes

$$\int_0^\infty y^2(t) + u^2(t)\,\mathrm{d}t$$

   over all stabilizing inputs.

   (b) Does the LQ problem with stability have a solution if $c = 0$? If not, show it does not. If yes, then show it and provide the optimal u (as function of x).

Hint: the Algebraic Riccati Equation is $A^{\mathrm{T}}P + PA + C^{\mathrm{T}}C - PBB^{\mathrm{T}}P = 0.$

6. Three questions.

   (a) Consider $\dot{x} = Ax$, $y = Cx$, $x(0) = x_0$, and suppose that $y(t)$ is constant iff $x_0 = 0$. Show that the system is observable and that A is invertible.

   (b) Let $A \in \mathbb{R}^{2\times 2}, b \in \mathbb{R}^{2\times 1}$ and consider the *discrete-time* system

$$x(k+1) = Ax(k) + bu(k), \qquad k \in \mathbb{Z}.$$

   Notice that the "time" k only takes integer values. The definition of controllability is essentially the same as that of continuous-time systems. Show that this system is controllable if $\begin{bmatrix} b & Ab \end{bmatrix}$ is invertible.

   (c) Determine an equivalent state representation of

$$\ddot{y}(t) - 4\dot{y}(t) + 3y(t) = 2\ddot{u}(t) + u(t).$$

problem:	1	2	3	4	5	6
points:	2+2+2+2+3	2	2+2+3	2+2	3+2	3+2+2

Exam grade: $1 + 9p/36$.