Systems Theory (202001355) – Exam 1

Date:	16-01-2023
Time:	8:45–11:45
Place:	SC1
Course coordinator:	Gjerrit Meinsma
Allowed aids during test:	a basic calculator

1. Let $b, c \in \mathbb{R}$ and consider the system described by

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} b \\ b \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 1 & c \end{bmatrix} x(t). \end{aligned} \tag{1}$$

   (a) Determine the transfer matrix from u to y, and determine all poles of the transfer function.

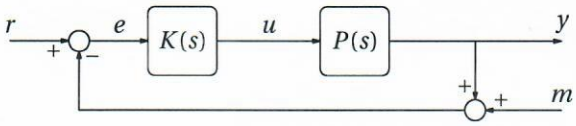
   (b) For which b is system stabilizable?

   (c) For which c is the system observable?

   (d) Now take $b = 1$. Determine a stabilizing state feedback $u(t) = -Fx(t)$ that places the two eigenvalues of the closed loop system at -1 and -2.

   (e) Now take $b = 1, c = 2$. Determine a stabilizing controller formed from an observer and state feedback.

2. Consider the system



   The signal $r(t)$ is a reference input, and $m(t)$ is a measurement noise. Usually reference inputs vary slowly with time, while measurement noise usually fluctuates a lot.

   (a) Determine the transfer function from m to y.

   For the rest of the problem assume that

$$K(s) = \frac{k}{s}, \qquad P(s) = \frac{1-s}{1+s}.$$

   (b) Determine all $k \in \mathbb{R}$ for which the closed loop is asymptotically stable.

   (c) Let $r(t) = 20\mathbb{1}(t)$ and $m(t) = 0$, and suppose the closed loop is asymptotically stable. Determine $\lim_{t\to\infty} y(t)$.

   (d) Let $r(t) = 0$ and $m(t) = \cos(100t)$, and suppose the closed loop is asymptotically stable. Argue that as $t \to \infty$ the signal $y(t)$ is very small.

3. What is the definition of a *BIBO-stable* linear system $y = \mathscr{H}(u)$? You may assume that $u, y : \mathbb{R} \to \mathbb{R}$.

4. Let $u, y : \mathbb{R} \to \mathbb{R}$. Consider the system $y = \mathscr{H}(u)$ defined by

$$y(t) = \int_{t-1}^{t+1} \tau u(\tau)\,\mathrm{d}\tau.$$

   (a) Is this system linear?

   (b) Is this system time invariant?

5. *LQ control.* Let $C \in \mathbb{R}$. Consider the scalar system

$$\dot{x}(t) = 1.5x(t) + u(t), \quad y(t) = Cx(t), \quad x(0) = x_0$$

and standard LQ cost $\int_0^\infty y^2(t) + u^2(t)\,\mathrm{d}t$.

   (a) Suppose now that $C = 2$. Determine the input u (as a function of x) that minimizes the cost over all stabilizing inputs, and determine the minimal cost. *Hint: the Algebraic Riccati Equation is* $A^{\mathrm{T}}P + PA + C^{\mathrm{T}}C - PBB^{\mathrm{T}}P = 0$.

   (b) Suppose now that $C = 0$. Determine the input u (as a function of x) that minimizes the cost over all stabilizing inputs, and explain why it is obvious that the minimal cost now is smaller than the minimal cost in part (a).

6. Three questions.

   (a) Determine an observable system $\dot{x}(t) = Ax(t) + Bu(t), y(t) = Cx(t)$ such that for every row C_i of C the reduced system $\dot{x}(t) = Ax(t) + Bu(t), y_i(t) = C_i x(t)$ is not observable.

   (b) Determine a Kalman controllability decomposition of

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u,$$
$$y = \begin{bmatrix} 1 & 2 \end{bmatrix} x.$$

   (c) Determine an equivalent state representation of

$$\ddot{y}(t) + 2\dot{y}(t) + 3y(t) = \ddot{u}(t) - u(t).$$

problem:	1	2	3	4	5	6
points:	2+2+2+2+2	2+2+2+2	2	2+2	2+3	2+3+2

Exam grade: $1 + 9p/36$.