Systems Theory RESIT (Module 2 — code: 202200238)

Date:	06-04-2023
Time:	08:45–11:45 (till 12:30 for students with special rights)
Place:	NH 115
Course coordinator:	Gjerrit Meinsma
Allowed aids during test:	None

1. Consider the differential equation

$$y^{(2)}(t) + 4y^{(1)}(t) + 4y(t) = u(t).$$

(a) Is the differential equation asymptotically stable?

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(b) Determine all real homogeneous solutions $y(t)$.

(c) Now let $u(t) = \mathrm{e}^{-t} + 2$. Determine the solution $y(t)$ of the above DE that satisfies $y(0) = 2, y^{(1)}(0) = 0$.

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2. Use the Routh-Hurwitz test to prove that a third order DE $y^{(3)}(t) + p_2 y^{(2)}(t) + p_1 y^{(1)}(t) + p_0 y(t) = 0$ is asymptotically stable iff $p_2, p_1, p_0 > 0$ and $p_2 p_1 > p_0$.

3. Consider this graph of some function $y(t)$:

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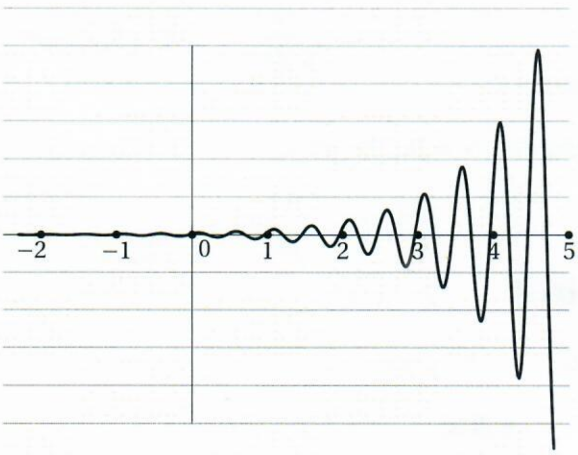

Determine a second order DE $\ddot{y}(t) + b\dot{y}(t) + cy(t) = 0$ whose solution $y(t)$ might be the one shown here. (I.e. determine $b, c \in \mathbb{R}$). Your estimate of b, c does not have to be very precise. But explain how you determine your b, c.

4. Let

$$A = \begin{bmatrix} -4 & 1 \\ -6 & 1 \end{bmatrix}.$$

(a) Determine e^{At}.

(b) Is $\dot{x}(t) = Ax(t)$ asymptotically stable?

(c) Let $\dot{x}(t) = Ax(t)$. Determine $x(0)$ such that $x(1) = \left[\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right]$.

5. Let $\alpha \in \mathbb{R}$. Consider

$$\dot{x} = \begin{bmatrix} 4 & \alpha \\ -6 & -5 \end{bmatrix} x + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u,$$
$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} x.$$

(a) For which α's is the system controllable?

(b) For which α's is the system observable?

(c) Is the system stabilizable if $\alpha = 3$?

(d) Take $\alpha = 0$. Determine a stabilizing state feedback $u = -Fx$.

(e) Take $\alpha = 0$. Determine a controller (with input y and output u) that stabilizes the system.

6. Determine a state representation $\dot{x}(t) = Ax(t) + Bu(t)$, $y(t) = Cx(t) + Du(t)$ of the differential equation

$$y^{(2)}(t) - 3y^{(1)}(t) + y(t) = u^{(1)}(t).$$

7. Three questions.

(a) Determine the reachable subspace of the system

$$\dot{x}(t) = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t),$$
$$y(t) = \begin{bmatrix} 1 & 2 \end{bmatrix} x(t).$$

(b) True or false: a system

$$\dot{x} = Ax + Bu, \qquad y = Cx + Du$$

is controllable if-and-only-if the system

$$\dot{\tilde{x}} = A^{\mathrm{T}}\tilde{x} + C^{\mathrm{T}}\tilde{u}, \qquad \tilde{y} = B^{\mathrm{T}}\tilde{x} + D\tilde{u}$$

is observable. [Prove the claim or provide counterexample.]

(c) What is the definition of *time constant* of a stable DE $\dot{y}(t) + cy(t) = u(t)$?

opgave:	1	2	3	4	5	6	7
punten:	1+2+4	2	2	4+1+2	2+2+2+2+2	2	2+2+2

Exam grade: $1 + 9p/36$.