Systems Theory (Module 2 — code: 202200238)

Date:	26-01-2024
Time:	13:45–16:45 (till 12:30 for students with special rights)
Place:	SC1
Course coordinator:	Gjerrit Meinsma
Allowed aids during test:	None

1. Consider the differential equation

$$y^{(2)}(t) + 2y^{(1)}(t) + 5y(t) = u(t).$$

   1pt (a) Is the differential equation asymptotically stable?

   2pt (b) Determine all real homogeneous solutions $y(t)$.

   4pt (c) Now let $u(t) = 10t - 1$. Determine all real solutions $y(t)$ of the above DE that satisfy $y(0) = 0$.

2pt 2. Suppose a 3rd-order DE has characteristic polynomial $(\lambda+3)(\lambda^2+4\lambda+5)$. What is its time constant?

3. Let $t \in \mathbb{R}$ and

$$A = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix}.$$

   4pt (a) Determine e^{At}.

   1pt (b) Is $\dot{x}(t) = Ax(t)$ asymptotically stable?

   2pt (c) Let $Q = A^{10}$. Determine e^{Qt}.

2pt 4. Determine a state representation $\dot{x}(t) = Ax(t) + Bu(t)$, $y(t) = Cx(t) + Du(t)$ of the differential equation

$$y^{(2)}(t) - 3y^{(1)}(t) + y(t) = u^{(2)}(t) - 4u^{(1)}(t).$$

5. Let $\beta \in \mathbb{R}$. Consider

$$\dot{x}(t) = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ \beta \end{bmatrix} u(t),$$
$$y(t) = \begin{bmatrix} 1 & \beta \end{bmatrix} x(t).$$

2pt (a) For which β's is the system controllable?

2pt (b) For which β's is the system observable?

2pt (c) For which β's is the system stabilizable?

2pt (d) Take $\beta = 0$. Determine a stabilizing state feedback $u = -Fx$.

2pt (e) Take $\beta = 0$. Determine an observer with both observer poles equal to -1.

2pt (f) Take $\beta = 0$. Is there a static output feedback $u = -Hy$ (with $H \in \mathbb{R}$) that stabilizes the system?

6. Three questions.

2pt (a) Determine the unobservable subspace of the system

$$\dot{x}(t) = \begin{bmatrix} -2 & 2 \\ -1 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(t),$$
$$y(t) = \begin{bmatrix} -1 & 1 \end{bmatrix} x(t).$$

2pt (b) Is there an $A \in \mathbb{R}^{2\times 2}$ such that

$$e^{At} = \begin{bmatrix} e^{-t} & te^{t} \\ 0 & e^{t} \end{bmatrix}?$$

2pt (c) What is the definition of a *detectable* system?

problem:	1	2	3	4	5	6
points:	1+2+4	2	4+1+2	2	2+2+2+2+2+2	2+2+2

Exam grade: $1 + 9p/36$.