Systems Theory (202001355) – Exam (resit)

Date:	26-01-2024
Time:	13:45–16:45 (till 17:30 for students with special rights)
Place:	SC1
Course coordinator:	Gjerrit Meinsma
Allowed aids during test:	a basic calculator

1. Consider

$$\dot{x} = \begin{bmatrix} 0 & 4 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u,$$
$$y = \begin{bmatrix} 1 & -1 \end{bmatrix} x.$$

   (a) Is this system controllable?

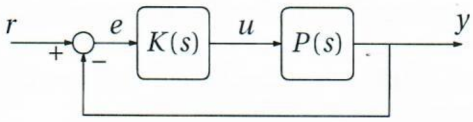
   (b) Determine the reachable subspace.

   (c) Determine a feedback $u = -Fx$ that makes $A - BF$ asymptotically stable.

   (d) Determine an observer with observer poles -1 and -2.

   (e) Determine a controller (with input y and output u) that stabilizes the closed loop system.

2. Consider the standard closed-loop system



and assume that the system $P(s)$ (with input u and output y) is described by the differential equation $\ddot{y} - 4y = \dot{u} + 2u$.

   (a) Determine the transfer function $P(s)$ (from u to y) from the differential equation.

   (b) Given this $P(s)$ prove that a controller $K(s)$ of the form $K(s) = \frac{N_K(s)}{D_K(s)}$ (with $N_K(s), D_K(s)$ polynomial) stabilizes the closed loop if-and-only-if

$$K(s) = \frac{Q(s) - (s-2)D_K(s)}{D_K(s)}$$

   for some asymptotically stable polynomial $Q(s)$.

   (c) Given this $P(s)$ determine a stabilizing controller $K(s) = \frac{N_K(s)}{D_K(s)}$ such that

   - $H_{y/r}(s)$ has DC-gain equal to 1,
   - $K(s)$ is *proper* (meaning that the degree of polynomial $D_K(s)$ is larger than or equal to the degree of polynomial $N_K(s)$).

3. Write the differential equation $\ddot{y} - 4y = \dot{u} + 2u$ in state space form $\dot{x} = Ax + Bu, y = Cx + Du$.

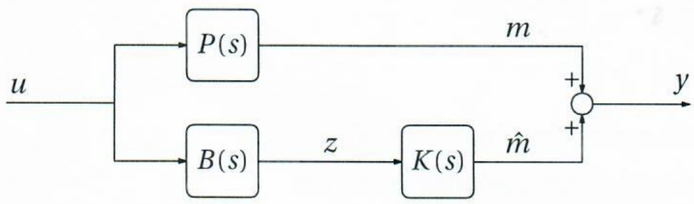
4. Consider the BIBO, LTI system $y = \mathscr{H}(u)$ with frequency response

$$H(\mathrm{i}\omega) = \frac{1 + \mathrm{e}^{-\mathrm{i}\omega}}{\mathrm{i}\omega + 10}.$$

For which $\alpha \in \mathbb{R}$ does the response $y(t)$ to $u(t) = \cos(\alpha t)\mathbb{1}(t)$ have the property that $\lim_{t\to\infty} y(t) = 0$?

5. Three questions.

   (a) Determine the transfer matrix from u to y for



   Your answer must be valid for the case that the signals contain more than one component (i.e. the transfer matrices might indeed be matrices).

   (b) Formulate the *Kalman Controllability Decomposition* theorem for systems of the form $\dot{x} = Ax + Bu$.

   (c) What is the definition of *LTI system* for systems of the form $y = \mathscr{H}(u)$?

6. *LQ control.* Let $a \in \mathbb{R}$ and consider the scalar system

$$\dot{x}(t) = ax(t) + 2u(t), \quad y(t) = 2x(t), \quad x(0) = x_0$$

and standard LQ cost $\int_0^\infty y^2(t) + u^2(t)\,\mathrm{d}t$.

   (a) Determine the input $u(t)$ — as a function of $x(t)$ and a — that minimizes the LQ cost over all stabilizing inputs, and determine optimal cost and the eigenvalue of the corresponding closed-loop system. *Hint: the Algebraic Riccati Equation is $A^{\mathrm{T}}P + PA + C^{\mathrm{T}}C - PBB^{\mathrm{T}}P = 0$.*

   (b) For which *two* values of $a \in \mathbb{R}$ does the closed-loop system have eigenvalue -5?

   (c) What happens with the optimal cost as $a \to -\infty$, and explain in words why this makes sense.

Problem:	1	2	3	4	5	6
Points:	2+2+3+2+2	2+3+2	2	2	2+3+3	3+1+2

Exam grade: $1 + 9p/36$.