Kenmerk: EWI2019/TW/DMMP/MU/Mod7/Exam2

Exam 2, Module 7, Codes 201400483 & 201800141

Discrete Structures & Efficient Algorithms

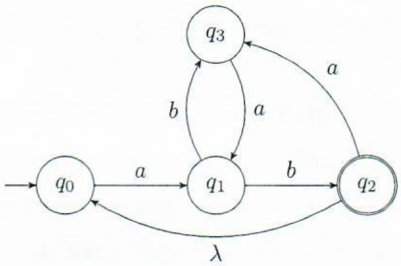
Friday, March 29, 2019, 13:45 - 15:45

All answers need to be motivated. No calculators. You are allowed to use a handwritten cheat sheet (A4, both sides).

This second exam of Module 7 consists of the **Languages & Machines part** only, and is a **2h exam**. The total is 50 points. Your exam grade is the maximum of 1 and the total number of points divided by 5, rounded to one digit.

Languages & Machines

1. (11 points) Consider the following NFA with λ-steps M (only q_2 is accepting):



   (a) First eliminate state q_3, adding new transitions labeled with regular expressions, to preserve the accepted language. Show the resulting "expression graph".

   (b) Continue the construction, by eliminating q_1 as well, and read off a regular expression E with $\mathcal{L}(E) = \mathcal{L}(M)$.

   (c) Provide the λ-closure and input-transition function of the automaton M in a table.

   (d) Transform the automaton M in a systematic manner to a (possibly incomplete) DFA.

2. (9 points) Consider the definitions of the following languages over $\Sigma = \{a, b\}$:

   - Language $L_1 := \{a^{3i}\, b^{j+1} \mid 0 \leq i \text{ and } 0 \leq j\}$
   - Language $L_2 := \{b^i\, a^j\, b^i \mid 0 \leq i \text{ and } 0 \leq j\}$
   - Language L_3 is the set of words over Σ that can be found in the Oxford English Dictionary.
   - Language L_4 is an (arbitrary) *infinite* language

   Indicate for each of the following languages if they are regular or not. Motivate your answers, either by a proof or a construction.

   (a) Language L_1

   (b) Language L_2

   (c) Language $\overline{L_4} \cap L_3$

3. (5 points) Which variables can be derived by chain rules from C in the following grammar G_1? Provide an equivalent grammar without chain rules.

$$G_1 = \begin{cases} S & \to & A\,B \mid C \\ A & \to & a\,A \mid B \\ B & \to & b\,B \mid C \\ C & \to & c\,C \mid a \mid A \end{cases}$$

4. (5 points) Provide a *regular* grammar, equivalent to the following context-free grammar G_2:

$$G_2 = \begin{cases} S & \to & B\,S \mid A\,B \\ A & \to & a\,A\,a \mid \lambda \\ B & \to & b \mid B\,b \end{cases}$$

5. (5 points) Consider the context-free language $L = \{a^i\,b^j \mid i > j > 0\}$.
Give a PDA (pushdown automaton) for L. Provide a *short* explanation.

6. (9 points, every wrong answer costs 2 points) Indicate for each of the following statements if they are TRUE or FALSE. (No explanation required).

   (a) Every contextfree grammar (CFG) has a Turing Machine (TM) accepting the same language.

   (b) Every contextfree grammar (CFG) has an equivalent extended PDA with two states.

   (c) The class of contextfree languages is closed under complement.

   (d) The class of contextfree languages is closed under union.

   (e) To every PDA there exists an equivalent deterministic PDA.

   (f) To every TM there exists an equivalent deterministic TM.

   (g) The language of (encoded) terminating Turing Machines is not recursive, but it is recursively enumerable.

   (h) Given a grammar G in Chomsky Normal Form and a word w, one can decide in polynomial time whether $w \in \mathcal{L}(G)$.

   (i) The class of recursive languages is closed under complement.

7. (6 points) Which language is *decided* by the following Turing Machine (only q_3 is accepting)? Explain your answer *shortly*.