

Re-Exam 2, Module 7, Codes 201700304 & 201800141

Discrete Structures & Efficient Algorithms

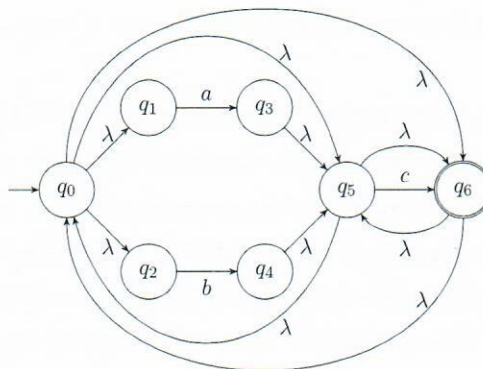
Wednesday, April 17, 2019, 8:45 - 10:45

All answers need to be motivated. No calculators. You are allowed to use a handwritten cheat sheet (A4, both sides).

This second re-exam of Module 7 consists of the **Languages & Machines part** only, and is a **2h exam**. The total is 50 points. Your exam grade is the 1 plus the total number of points multiplied by $\frac{9}{50}$, rounded to one digit.

Languages & Machines

1. (11 points) Consider the following NFA- λ , M (only q_6 is accepting):



- (a) M was constructed systematically from regular expression E (some λ -steps are omitted). What was the original expression E ?
- (b) Give a table with the λ -closure and input-transition function of M .
- (c) Transform the automaton M systematically to an (incomplete) DFA.
- (d) Give the minimal DFA for the same language. Just provide the answer, you don't need to provide a construction.
2. (9 points) Consider the definitions of the following languages over $\Sigma = \{a, b\}$:
- Language $L_1 := \{b^{3i} a c^i \mid 0 \leq i\}$
 - Language $L_2 := \{b^j a^i b^j \mid 0 \leq i \leq 1024 \text{ and } 0 \leq j \leq 3i\}$
 - Language L_3 is an (arbitrary) *regular* language
 - Language L_4 is an (arbitrary) *context-free* language

Indicate for each of the following languages if they are regular or not. Motivate your answers, either by a proof or a construction.

- (a) Language L_1
- (b) Language L_2
- (c) Language $\overline{L_3} \cup L_4$

3. (7 points) Transform the following context-free grammar G step by step to Chomsky Normal form. Specify clearly which steps you take, and what the intermediate results are:

$$G = \begin{cases} S \rightarrow ABC \mid \lambda \\ A \rightarrow aA \mid a \\ B \rightarrow bB \mid A \\ C \rightarrow cC \mid \lambda \end{cases}$$

4. (8 points) Consider the context-free language $L = \{a^{2i} b^j \mid j \geq i > 0\}$. Give a DPDA (deterministic pushdown automaton) for L . Provide a *short* explanation.
5. (9 points, every correct answer gives you 1 point, every wrong answer costs 2 points, not answering a question gives 0 points for it. The total number of points awarded for this exercise is at least 0)
Indicate for each of the following statements if they are TRUE or FALSE. (No explanation required).

- (a) Every regular grammar has a Turing Machine (TM) accepting the same language.
- (b) Every PDA has an equivalent extended PDA with two states accepting the same language.
- (c) Every PDA has an equivalent deterministic PDA accepting the same language.
- (d) The class of contextfree languages is closed under complement.
- (e) The class of regular languages is closed under union.
- (f) To every Turing Machine (TM) there exists an equivalent deterministic TM.
- (g) The language of (encoded) terminating Turing Machines is not recursive, but it is recursively enumerable.
- (h) The class of recursive languages is closed under complement.
- (i) For every language L there is a Turing Machine (TM) deciding L .

6. (6 points) Which language is *decided* by the following Turing Machine (only q_3 is accepting)? Explain your answer *shortly*.

