Exam 2, Module 7, Codes 201700304 & 201800141

Discrete Structures & Efficient Algorithms

Friday, April 3, 2020, 8:45 - 10:45

All answers need to be motivated.

This second exam of Module 7 consists of the **Languages & Machines part** only, and is a **2h exam**. The total is 50 points. Your exam grade is 1 plus the total number of points multiplied by $\frac{9}{50}$, rounded to one digit. You are allowed to use the textbook(s), lecture slides, as well as a handwritten cheat sheet during the exam.

Please read the following paragraph carefully, and **copy the text below** (including at the end your name, student number, location, date, and signature) to your answer sheet.

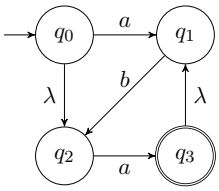
By testing you remotely in this fashion, we express our trust that you will adhere to the ethical standard of behaviour expected of you. This means that we trust you to answer the questions and perform the assignments in this test to the best of your own ability, without seeking or accepting the help of any source that is not explicitly allowed by the aforementioned conditions of this test.

Text to be copied:

I will make this test to the best of my own ability, without seeking or accepting the help of any source not explicitly allowed by the conditions of the test.
[Name, Student no., Location, Date, Signature].

Languages & Machines

1. (8 points) Consider the following NFA-λ, M (only q_3 is accepting):



   (a) Transform the automaton M step by step to an expression.

   (b) Give a table with the λ-closure and input-transition function of M.

   (c) Transform the automaton M systematically to an DFA.

2. (8 points) Consider the definitions of the following languages over $\Sigma = \{a, b\}$:

   - Language $L_1 := \{a^{i+2}\, c\, b^i \mid 0 \leq i\}$
   - Language $L_2 := \{b^j\, c^i\, b^j\, (cb)^i \mid 0 \leq i \leq 42 \text{ and } 0 \leq j \leq 3i\}$
   - Language L_3 is an (arbitrary) *regular* language
   - Language $L_4 = L(M)$ for an (arbitrary) *deterministic* pushdown automaton M.

   Indicate for each of the following languages if they are regular or not. Motivate your answers, either by a proof or a construction.

   (a) Language L_1
   (b) Language L_2
   (c) Language $\overline{L_3} \cup L_4$

3. (5 points) Consider the language $L := \{c^j\, a^i\, c^j\, b^i \mid 0 \leq i \text{ and } 0 \leq j \leq 3i\}$. Prove without using the pumping lemma that L is not regular.

4. (10 points) Consider the following context-free grammar G:

$$G = \begin{cases} S & \to & A\,C\,B \mid AC \mid \lambda \\ A & \to & a\,A \mid a \\ B & \to & b\,B \mid C \\ C & \to & c\,C \mid \lambda \end{cases}$$

   (a) Transform G step by step to a grammar G' in Chomsky Normal form. Specify clearly which steps you take, and what the intermediate results are.
   (b) Let $w = aacc$. Apply the CYK-algorithm (after Cocke-Younger-Kasami) to decide whether $w \in L(G')$. Provide a derivation tree for w as well.

5. (8 points) Consider the context-free language $L = \{a^{i+1}\, b^{2i} \mid i \geq 0\}$.

   (a) Give a context-free grammar in Greibach Normal form for L
   (b) Give an (extended) PDA with at most two states for L. Provide a *short* explanation.

6. (5 points) Prove or disprove: the language $L := \{b^j\, a^i\, c^j\, b^i\, d^j \mid 0 \leq j \text{ and } 0 \leq i \leq 3j\}$ is context-free.

7. (6 points) Which language is *decided* by the following Turing Machine (only q_3 is accepting)? Explain your answer *shortly*.