Uitwerking Tentamen Calculus I - 27-11-98

1 (α) $\sum_{k=1}^{1} \frac{1}{k(k+1)} = \frac{1}{1(1+1)} = \frac{1}{1+1}$ $\square$

(β)

$$\sum_{k=1}^{n+1} \frac{1}{k(k+1)} = \frac{n}{n+1} + \frac{1}{(n+1)(n+2)} = \frac{n(n+2)+1}{(n+1)(n+2)} = \frac{(n+1)^2}{(n+1)(n+2)} = \frac{n+1}{n+1+1}$$

2 (a) ..

$$\left(1+\frac{1}{n+1}\right)^{n+1} \leq (1+x)^{n+1} \leq \left(1+\frac{1}{n-1}\right)^{n+1}$$

$$= \left(1+\frac{1}{n-1}\right)^{n-1} \cdot \left(1+\frac{1}{n-1}\right)^{2}$$

$\downarrow e$ $\qquad \downarrow e \qquad \downarrow 1$

$\downarrow$

$e.$

3a lim a. $x<1$ formule

$x>1$ formule

$$f(1)=1+1^2=2 \qquad \lim_{x\uparrow 1} f(x) = \lim_{x \uparrow 1} 1+x^2 = 2$$

$$\lim_{x\downarrow 1} f(x) = \lim_{x\downarrow 1} 1+\frac{1}{x^2} = 2$$

$\Rightarrow$ f cont in 1

b buiten $x=1$ formulefunctie

c $f'_r(1) = \lim_{x\downarrow 1} \frac{\left(1+\frac{1}{x^2}\right)-2}{x-1} = \lim_{x\downarrow 1} \frac{\frac{1}{x^2}-1}{x-1} = \lim_{x\downarrow 1} \frac{\frac{(1-x)(1+x)}{x^2}}{x-1}$

$= \lim_{x\downarrow 1} -\frac{(1+x)}{x^2} = -2$

$f'_l(1) = \lim_{x\uparrow 1} \frac{1+x^2-2}{x-1} = \lim_{x\uparrow 1} \frac{(x-1)(x+1)}{x-1} = 2$ $\qquad$ f niet diff in 1

4 $e^{2x} = 1 + 2x + \frac{4x^2}{2!} + o(x^2)$

$\cos x = 1 - \frac{x^2}{2!} + o(x^2)$

$\ln(1+x) = x - \frac{x^2}{2} + o(x^2)$

$$\lim_{x\to 0} \frac{e^{2x} - \cos x - 2\ln(1+x)}{x^2} = \lim_{x\to 0} \frac{1 + 2x + \frac{x^2}{2} + o(x^2) - 1 + \frac{x^2}{2} + o(x^2) - 2x + x^2 + o(x^2)}{x^2}$$

$$\lim_{x\to 0} \frac{2x^2 + o(x^2)}{x^2} = 2$$

5a $\int (x^2+1)\ln x \, dx = \left(\frac{1}{3}x^3 + x\right)\ln x - \int \left(\frac{1}{3}x^3 + x\right)\frac{1}{x}\,dx$

$= \left(\frac{1}{3}x^3 + x\right)\ln x - \frac{1}{9}x^3 - x + C$

b1 $\int \frac{e^x \, dx}{e^x + e^{-x}}$ $\quad e^x = u \quad x = \ln u \quad \frac{dx}{du} = \frac{1}{u}$

$$\int \frac{u}{u + \frac{1}{u}} \cdot \frac{1}{u}\,du = \int \frac{u\,du}{u^2+1} = \frac{1}{2}\ln(e^{2x}+1) + C$$

$$\int_{-\infty}^{0} \frac{e^x}{e^x + e^{-x}}\,dx = \lim_{y\to -\infty} \frac{1}{2}\ln(e^{2x}+1)\Big|_y^0 = \frac{1}{2}\ln 2$$

c $\frac{d}{dx}\int_e^{e^x} \frac{1}{\ln t}\,dx = e^x \cdot \frac{1}{\ln e^x} = \frac{e^x}{x}$

6a $x_1 = r\cos\varphi$ $\quad \lim_{r\to 0} \frac{r^4(3\cos^2\varphi\sin^2\varphi - 4\sin^4\varphi)}{r^2} = 0 = f(0,0) \Rightarrow$ cont

$x_2 = r\sin\varphi$

buiten $(x_1, x_2) = (0,0)$ geen problem.

b ..

c. $\partial_1 f = \lim_{t\to 0} \frac{f(t,0) - f(0,0)}{t} = \lim_{t\to 0} \frac{0-0}{t} = 0$

$\partial_2 f = \lim_{h\to 0} \frac{f(0,h) - f(0,0)}{h} = \lim_{h\to 0} \frac{\frac{-4h^4}{h^2} - 0}{h} = -4$. ~~bestaat niet~~

d $\lim_{x\to 0} \frac{\rho(x)}{|x|} = \lim_{x\to 0} \frac{f(x) - (-4)x_2}{|x|}$ bestaat niet. (dus!)