Uitwerking Tentamen Calculus I
Vrijdag 10-11-2006 CT/TN/EL

1a $\displaystyle\lim_{x\to 0} \frac{9(2x+1)^{\frac{1}{3}} - 7\cos(x) - 2e^{3x}}{\sin(x^2)}$

type $\frac{0}{0}$ $= \displaystyle\lim_{x\to 0} \frac{f(x)}{g(x)}$

$f'(x) = 9(2x+1)^{-\frac{2}{3}} \cdot \frac{1}{3} \cdot 2 + 7\sin x - 6e^{3x}$

$g'(x) = \cos(x^2) \cdot 2x$ $\qquad \displaystyle\lim_{x\to 0} \frac{f'(x)}{g'(x)} =$ type $\frac{0}{0}$

$$\lim_{x\to 0} \frac{f''(x)}{g''(x)} = \lim_{x\to 0} \frac{9(2x+1)^{-\frac{5}{3}} \cdot \frac{1}{3} \cdot 2 \cdot -\frac{2}{3} \cdot 2 + 7\cos x - 18e^{3x}}{\cos(x^2) \cdot 2 - \sin(x^2) \cdot (2x)^2}$$

$$= \frac{9 \cdot \frac{-8}{9} + 7 - 18}{2} = -\frac{19}{2} \Rightarrow \square$$

b $\displaystyle\lim_{t\to 0} \frac{\sqrt{9+\sin t} - 3}{t}$: dit is de definitie van afgeleide van $\sqrt{9+\sin x}$ in $x=0$

dus $f'(x) = \frac{1}{2}(9+\sin x)^{-\frac{1}{2}} \cdot \cos x$: $f'(0) = \frac{1}{6}$

2 $f(x) = (x+7)^{1/3}$ $\qquad f(1) = 2$ $\qquad f(1.1) = 2{,}0082988$

$f'(x) = (x+7)^{-2/3} \cdot \frac{1}{3}$ $\qquad f'(1) = \frac{1}{4.3} = \frac{1}{12}$

$f''(x) = (x+7)^{-5/3} \cdot -\frac{2}{3} \cdot \frac{1}{3}$ $\qquad f''(1) = \frac{1}{32.} \cdot -\frac{2}{3} \frac{1}{3} = \frac{-1}{16.9}$

$f'''(x) = (x+7)^{-8/3} \cdot \frac{-5}{3} \cdot \frac{-2}{3} \cdot \frac{1}{3}$

a $T_2(x) = 2 + \frac{1}{12}(x-1) + \frac{1}{2!} \cdot \frac{-1}{16.9}(x-1)^2$

b $T_2(1.1) = 2 + \frac{1}{12}(0.1) + \frac{1}{2!} \cdot \frac{-1}{16.9}(0.1)^2 = \cancel{2.01\ldots 2644}\ldots$

$2.00829 86$

c $\left|T_2(1.1) - f(1.1)\right| \leq \dfrac{f'''(1)}{3!}(1.1-1)^3 \Rightarrow 24.10^{-8}$

3a $\displaystyle\int \frac{1}{x^2+2x+5}\,dx = \int \frac{1}{(x+1)^2+4}\,dx \,\Big|_{x+1=u} = \int \frac{1}{u^2+4}\,du$

$u = 2z \Rightarrow \displaystyle\int \frac{1}{4z^2+4}\cdot 2dz = \frac{1}{2}\arctan(z) + C$

$= \frac{1}{2}\arctan\left(\frac{x+1}{2}\right) + C$

3b $\displaystyle\frac{d}{dx}\int_{x+1}^{x^2} \frac{1}{(u^2+1)^2}\,du \qquad f(u) = \frac{1}{(u^2+1)^2}$

$F(u): F'(u) = f(u)$

$\displaystyle = \int_{x+1}^{x^2} \frac{1}{(u^2+1)^2}\,du = F(x^2) - F(x+1)$

$\frac{d}{dx}\left(\dots\right) = F'(x^2).2x - F'(x+1).1$ en invullen

$F'(x^2) = \dfrac{1}{(x^4+1)^2} \qquad F'(x+1) = \dfrac{1}{((x+1)^2+1)^2}$

4a $z = 2-2i \qquad |z| = \sqrt{4+4} = 2\sqrt{2}$
$z = 2\sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right) = 2\sqrt{2}\left(\cos -\frac{\pi}{4} + i\sin -\frac{\pi}{4}\right)$

$\omega = -1 + i\sqrt{3} \quad |\omega| = \sqrt{1+3} = 2$
$\omega = 2\left(-\frac{1}{2} + \frac{1}{2}i\sqrt{3}\right) = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

b $z^5\omega^3 = (2\sqrt{2})^5(2)^3\left(\cos\left(-\frac{5}{4}\pi + \cancel{2\pi}\right) + i\sin\left(-\frac{5}{4}\pi + \cancel{2\pi}\right)\right)$

$\omega^6 = 2^6(\cos 4\pi + i\sin 4\pi) = 2^6$

5 $y'' + y = 2xe^{x} + 2\cos x$

<u>hom</u> $y'' + y = 0 \quad r^2 + 1 = 0 \quad r = \pm i \Rightarrow$

$y(x) = A\cos x + B\sin x$

<u>inh</u> y_{p_1} $y'' + y = 2xe^{x}$

$y_{p_1}(x) = (\alpha x + \beta)e^{x}$

$y_{p_1}'(x) = \alpha e^{x} + (\alpha x + \beta)e^{x}$

$y_{p_1}''(x) = \alpha e^{x} + \alpha e^{x} + (\alpha x + \beta)e^{x}$

$\Rightarrow 2\alpha e^{x} + (\alpha x + \beta)e^{x} + (\alpha x + \beta)e^{x} = 2xe^{x}$

$\Rightarrow 2\alpha = 2 \qquad \Rightarrow \alpha = 1 \qquad \Rightarrow y_{p_1}(x) = (x-1)e^{x}$

$2\alpha + 2\beta = 0 \qquad \beta = -1$

inh y_{p_2} $y'' + y = 2\cos x$

$y_{p_2}(x) = Ax\cos x + Bx\sin x$

$y_{p_2}'(x) = A\cos x + B\sin x - Ax\sin x + Bx\cos x$

$y_{p_2}''(x) = -A\sin x + B\cos x - A\sin x + B\cos x - Ax\cos x - Bx\sin x$

$\Rightarrow \underbrace{-2A\sin x + 2B\cos x - Ax\cos x - Bx\sin x}_{y''} + \underbrace{Ax\cos x + Bx\sin x}_{y} = 2\cos x$

$\Rightarrow$

$-2A = 0 \quad \Rightarrow A = 0 \quad B = 1$

$2B = 2 \qquad \Rightarrow y_{p_2}(x) = x\sin x$

$y_{in}(x) = \alpha\cos x + \beta\sin x + (x-1)e^{x} + x\sin x$

$y_{in}(0) = \alpha - 1 \Rightarrow -1$ dus $\alpha = 0 \quad y'(0) = 1 \Rightarrow \beta = 1$ $\square$

6 $f(x,y)$ gegeven

a $\lim_{(x,y)\to(0,0)} f(x,y) = \left(\begin{array}{l} x = r\cos\varphi \\ y = r\sin\varphi \end{array}\right) = \lim_{r\to 0^+} \frac{r^{\cancel{4}\,2}\cos\varphi\sin^3\varphi}{\cancel{r^2}} = 0 = f(0,0)$

$\Rightarrow f$ continu in $(0,0)$

b $\frac{\partial f}{\partial y} = \frac{3xy^2}{x^2+y^2} + \frac{(-1)xy^3}{(x^2+y^2)^2}\cdot 2y$

c $\frac{\partial f}{\partial y}(0,0) = \lim_{h\to 0} \frac{f(0,h)-f(0,0)}{h} = \lim_{h\to 0} \frac{0-0}{h} = 0$ $\square$

eventueel rekenfouten?