Example Exam 1 Stochastic Models M8

Problem 1

A store is known for its bargains. The store has the habit of lowering the price of its bargains each day, to ensure that articles are sold fast. Assume that you spot an item on Wednesday (there is only one of it left) that costs 30 Euro and that you would like to buy for a friend as present for Saturday. You know that the price will be lowered to 25 Euro on Thursday when the item is not sold, and to l0 Euro on Friday. You estimate that the probability that the item will be available on Thursday equals 0.7. You further estimate that assuming that it is still available on Friday when it was available on Thursday equals 0.6. You are sure that the item will no longer be available on Saturday. When you postpone your decision to buy the item to either Thursday or Friday, and the item is sold, you will buy another item of 40 Euro as present for Saturday.

a) Formulate the problem as stochastic dynamic programming problem. Specify phases, states, decisions and the optimal value function.
b) Draw the decision tree for this problem.
c) Give the recurrence relations for the optimal value function.
d) What is the minimal expected amount that you will pay for your present, and what is the optimal decision on Wednesday?

Problem 2

Each day you own 0 or 1 stocks of certain commodity. The price of the stock is a stochastic process that can be modeled as a Markov chain with transition rates as follows

		day n+1	
		100	200
day n	100	0.5	0.5
	200	0.25	0.75

At the start of a day at which you own a stock you may choose to either sell at the current price, or keep the stock. At the start of a day at which you do not own stock, you may choose to either buy one stock at the current price or do nothing. You have initial capital of 200.

Your target is to maximize the discounted value of the profit over an infinite horizon, use discountfactor 0.8 (per day).

a) Define the states and give for each state the possible decisions.
b) Formulate the optimality equations.
c) Carry out the initialization and two additional iterations of value iteration.
d) Formulate the L.P.-model to solve this problem. Describe how you can obtain the optimal policy from the LP formulation.
e) Choose a stationary policy and investigate using the policy iteration algorithm whether or not that policy is optimal.
f) Give the number of stationary policies. Motivate your answer by using the definition of stationary policy.

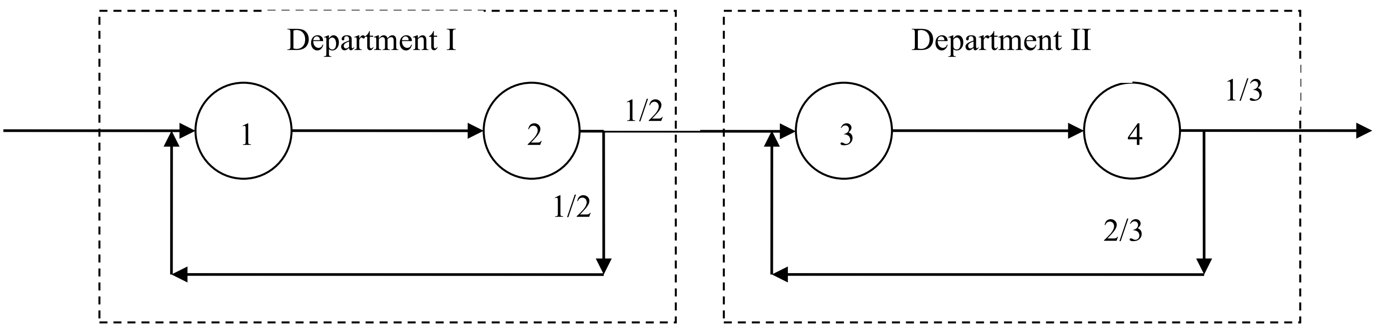
Problem 3
Consider a queueing system with 1 counter, to which groups of customers arrive according to a Poisson proces with intensity λ. The size of a group is 1 with probability p and 2 with probability 1-p. Customers are served one by one. The service time has exponential distribution with mean μ^{-1}. Service times are mutually independent and independent of the arrival process. The system may contain at most 3 customers. If the system is full upon arrival of a group, or if the system may contain only one additional customer upon arrival of a group of size 2, then all customers in the group are lost and will never return. Let $Z(t)$ record the number of customers at time t.

(a) Explain why $\{Z(t), t\geq 0\}$ is a Markov process and give the diagram of transitions and transition rates.
(b) Give the equilibrium equations (balance equations) for the stationary probabilities P_n, n=0,1,2,3.
(c) Compute these probabilities P_n, n=0,1,2,3.

The answers to the following questions may be provided in terms of the probabilities P_n (except for (h)).

(d) Give an expression for the average number of waiting customers.
(e) Give the departure rate and the rate at which customers *enter* the system.
(f) Give an expression for the average waiting time of a customer.
(g) What is the fraction of time the counter is busy?
(h) What is the average length of an idle period?
(i) Determine from (g) and (h) the average length of a period the system is occupied (= at least 1 customer in the system).
(j) What is the rate at which groups of size 2 *enter* the system?

Problem 4
Consider the open network in the following figure. The queueing system consists of 4 queues, 1, 2, 3 en 4. Queues 1 and 2 are department I, queues 3 and 4 are department II. The numbers at the arrows give the transition probabilities for customers routing among the stations, so a customer that leaves queue 4 routes to queue 3 with probability 2/3, and leaves the network with probability 1/3. Each station has a single server, and each customer arriving to a queue can enter. Service is in order of arrival. Service times have exponential distribution with means $1/\mu_1=1/4$, $1/\mu_2=1/3$, $1/\mu_3=1/2$, $1/\mu_4=1$. The arrival intensity to station 1 is γ_1 (Poisson). [Note: queue i refers to the system consisting of the waiting room plus the server, i=1,2,3,4.]



a) Formulate the traffic equations and solve these equations.
b) Give the stability condition?
c) Give the equilibrium distribution of the queue length at each of the stations 1, 2, 3 and 4.
d) Give the joint distribution of the queue lengths at the stations (product form).
e) Give for each station the average number of customers in the queue, and the average sojourn time of a customer at that queue.
f) Give an expression for the average sojourn time in Department II.