Example Exam 2 Stochastic Models M8

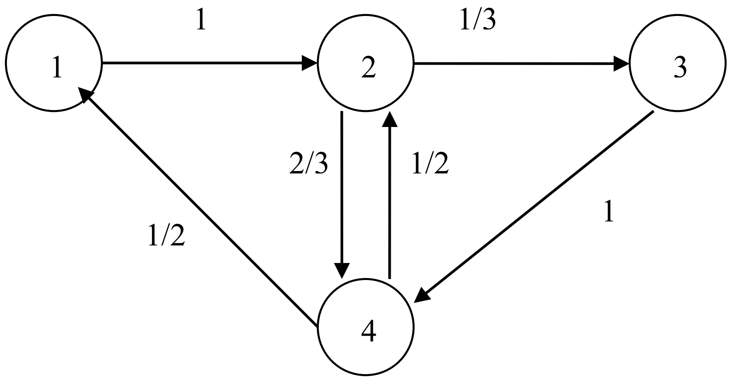
Problem 1

Consider the following variant of the M/M/1 queueing system. Upon completion of a customer's service, the customer again joins the (end of the) queue with probability p. Each customer continues to do so after each service until he or she has left the system. All service times are mutually independent exponentially distributed variables. Also, the probability p is independent of the number of services a customer has already completed.

(a) Draw the transition diagram for the number of customers in this queueing system.
(b) Give the balance equations and solve these.
(c) Give the stability condition for this problem.
(d) What is the probability that a customer undergoes exactly k services?
(e) What is the mean number of customers in the queue?
(f) What is the utilization of the server?
(g) Which classical M/M/1 queueing system has the same average number of customers in the queue as the system considered in this problem?

Problem 2

Consider the closed network of the following figure. The number at the arrows give the transition probabilities for a customer leaving the queue to route to a subsequent queue. Every station contains a single server, and all arriving customers may enter the station. Service is in order of arrival. The service times have an exponential distribution with: $\mu_1=4$, $\mu_2=3$, $\mu_3=2$, $\mu_4=1$.



(a) Give the joint stationary distribution for the number of customers in the four stations for m=1, 2, and 3 (m=total number of customers in the network).
(b) Obtain using Mean Value Analyse the average number of customers and the average sojourn time in the four queues for m=1, 2 and 3.
(c) Determine for m=1 the average number of transitions for a customer to return for the first time to station 1.

Problem 3
G. Ambler has € 10000 available for a second hand car, but would like to buy a fast car that costs € 25000. He needs the money for that car quickly, and would like to increase his capital to € 25000 via a gambling game. To this end, he can play a game in which he is allowed to toss an imperfect (with probability 0.4 for heads) coin three times. For each toss he may bet each amount (in multiples of € 1000 and the amount should be in his possession). He will win the amount (i.e., receives twice the amount of the bet) when he tosses head, and loses his betted amount when he tosses tails. Use stochastic dynamic programming to determine a strategy that maximises the probability of reaching € 25000 after three tosses.

(a) Formulate the problem as a stochastic dynamic program. What do you choose as stages n, states i, decisions d, and optimal value function?
(b) What is the recurrence relation of the optimal value function?
(c) Use dynamic programming to determine the optimal policy, and describe in words what this policy does. What is the expected probability of success?

Problem 4
The inventory of a certain good is inspected periodically. If a replenishment order is placed of size x>0 (integer), the ordering costs are 8+2x. The delivery time is zero. The demand is stochastic and equals 1 or 2 with probability ½. Demand in subsequent periods are independent. The size of a replenishment order must be such that (i) demand in a period is always satisfied, and (ii) the stock at the end of a period never exceeds 2. The holding costs in a period are 2 per unit of inventory remaining at the end of a period. The target is to minimize the expected discounted costs over an infinite horizon, using discount factor 0.8.

(a) Model this problem as a Markov decision problem. What do you choose as states, decisions and optimal value function?
(b) Determine the direct costs and transition probabilities for each state and decision.
(c) Formulate the corresponding optimality equations.
(d) Carry out the initialization and two additional iterations of the value iteration algorithm. What approximation to the optimal values do you find? What is the policy that corresponds to these values?
(e) Choose an ordering policy, and use the policy iteration algorithm to investigate whether or not this policy is optimal.

Suppose that, instead of minimizing the expected discounted costs, the objective would be to minimize the average costs.

(f) Formulate an LP-model that could be used to determine an average optimal policy. Also, describe how the average optimal policy may be obtained from the optimal solution of the LP.