Exam Limits to Computing (201300042)

Thursday, November 1, 2018, 8:45 – 11:45

- You can bring printouts of the sheets, lecture notes, exercises, solutions (mine and yours) to the exam or anything else printed or written on paper.
- Books and electronic devices of any kind are not allowed.
- This exam consists of five problems.
- Please start a new page for each problem.
- The total number of points is 50. The distribution of points is according to the following table.

1a: 8 1b: 7 1c: 3	2: 8	3: 6	4a: 4 4b: 6	5a: 2 5b: 2 5c: 2 5d: 2

1. Decidability and Recursive Enumerability

Let

$$\textsc{Disjoint} = \{\langle g, h\rangle \mid g, h \in \mathcal{G} \text{ and } \operatorname{dom}(\varphi_g) \cap \operatorname{dom}(\varphi_h) = \emptyset\}.$$

(a) *(8 points)* Is $\textsc{Disjoint} \in \mathsf{REC}$?

(b) *(7 points)* Is $\textsc{Disjoint} \in \mathsf{co\text{-}RE}$?

(c) *(3 points)* Is $\textsc{Disjoint} \in \mathsf{RE}$?

2. Complexity and NP-Completeness

(8 points) Prove the following statement: For every $\varepsilon > 0$, there exists a language L such that

- L is NP-complete and
- $L \in \mathsf{DTime}(2^{(n^\varepsilon)})$.

3. NP and co-NP

(6 points) Let

$$\textsc{SubgraphIsomorphism} = \{(G, H) \mid G \text{ and } H \text{ are undirected graphs and } G \text{ contains } H \text{ as an induced subgraph}\}.$$

Prove that $\textsc{SubgraphIsomorphism}$ is NP-complete.

4. Reductions and Complexity

We say that A is *quasi-linear-time reducible* to B (denoted by $A \leq_q B$) if there exists a constant $c \geq 0$ and a function f that is computable in time $O(n(\log n)^c)$ such that $x \in A$ if and only if $f(x) \in B$ for all x.

(a) *(4 points)* Show that $\leq_q$ is transitive.

(b) *(6 points)* Prove that there are no P-complete with respect to $\leq_q$. This means that there is no $L \in \mathsf{P}$ with $A \leq_q L$ for all $A \in \mathsf{P}$.

$A \leq_q L$

5. Questions

Are the following statements true or false? Justify your answers.

(a) *(2 points)* If $A \subseteq \mathbb{N}$ is an index set, then A or $\overline{A}$ are recursively enumerable.

(b) *(2 points)* If $\mathsf{NP} = \mathsf{NL}$, then $\mathsf{P} \subsetneq \mathsf{PSPACE}$.

(c) *(2 points)* For all sets $A, B \subseteq \mathbb{N}$, the following holds: If $A \cap B \in \mathsf{REC}$, then $A, B \in \mathsf{REC}$.

(d) *(2 points)* For all sets $A, B \subseteq \mathbb{N}$, the following holds: If $A \in \mathsf{REC}$ and $B \in \mathsf{RE}$, then $A \setminus B \in \text{co-}\mathsf{RE}$.