WORK OUT ON A PAPER SEPARATE FROM ANALYSIS

Test 2 Linear Optimization (course code 201300057)

Give motivations for all your answers.

1. (8 pt) Solve by the Simplex Method:

$$\begin{array}{lrrrl} \min & -x_1 & -2x_2 & -3x_3 & \\ \text{s.t.} & & x_2 & +2x_3 & \leq 5 \\ & x_1 & -x_2 & -x_3 & \leq 2 \\ & x_1 & & +x_3 & \leq 4 \\ & & \mathbf{x} \geq 0 & & \end{array}$$

2. (10 pt) For each of the following statements, determine whether it is true or false. Provide either a short argument of at most two lines, or a (counter)example.

   a) If a linear program is infeasible, the linear program used in phase 1 of the 2-phase simplex method to solve it, is also infeasible.

   b) Using the simplex method with lexicographic pivoting rule, in each step the cost strictly decreases.

   c) Consider a linear program in standard form. If we have 2 identical columns, corresponding to variables with different costs, then in any basic feasible solution that is optimal, at most one of these variables is basic.

   d) Consider a linear program in standard form that is infeasible, but which becomes feasible if the last equality is omitted. The dual of the original (infeasible) problem is feasible and the optimal cost is infinite.

   e) The linear programs below are duals of each other. (If true, no explanation is needed, if false, give the wrong constraint(s)).

$$\begin{array}{ll} \min & \mathbf{c}'\mathbf{x} \\ \text{s.t.} & A\mathbf{x} \geq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{array}$$

$$\begin{array}{ll} \max & \mathbf{p}'\mathbf{b} \\ \text{s.t.} & \mathbf{p}'A = \mathbf{c}' \\ & \mathbf{p} \leq \mathbf{0} \end{array}$$

Please turn over for the test of Analysis