Robust Control — EXAM

Course code:	191560671
Date:	12-04-2016
Time:	13:45–16:45 (till 17:30 for students with special rights)
Course coordinator & instructor:	G. Meinsma
Type of test:	open book
Allowed aids during the test:	printed lecture notes, basic calculator

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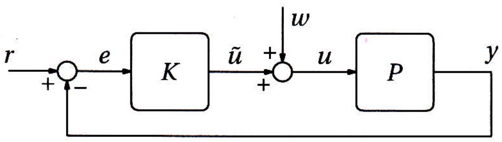
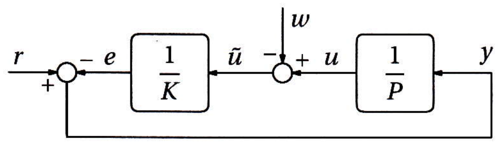
1. Determine the $\mathbb{H}_\infty$-norm of the 2×2 transfer matrix $\begin{bmatrix} \frac{1}{s+2} & 0 \\ 0 & \frac{2s}{s+4} \end{bmatrix}$.

2. There is a "fierce debate" between some control engineers and mathy behaviorists. For behaviorists these two configurations (I) and (II) are equivalent:



(I)



(II)

For control engineers they are not. Notice that the direction of most arrows in (II) are inverted and that the systems are inverted as well. We assume that $P(s)$ and $K(s)$ are rational SISO systems and that $P(s)$ and $K(s)$ are invertible.

   (a) Show that closed loop (I) is asymptotically stable iff closed loop (II) is asymptotically stable

   (b) Theorem 3.4.3 says that if $P \in \mathbb{H}_\infty$ then the closed loop is internally stable iff $\frac{K}{I+PK} \in \mathbb{H}_\infty$. The behaviorist would then immediately conclude that "if $\frac{1}{P} \in \mathbb{H}_\infty$ then the closed loop is internally stable iff

$$\frac{\frac{1}{K}}{1+\frac{1}{P}\frac{1}{K}} = \frac{P}{1+PK}$$

   is in $\mathbb{H}_\infty$". Show that she is right.

(It might seem that the behaviorists are right, but once delays enter the loop the story changes.)

3. Consider the system of Example 3.7.4. All parameters g, m, M, ℓ are positive. Explain why "Freudenberg-Looze" (Chapter 5) implies that this system is difficult to control. In particular address the problem for the case that $m/M \gg 1$ and for the case that $m/M \ll 1$.

4. *Chapter 6:* Consider

$$\begin{bmatrix} \dot{x} \\ z \\ y \end{bmatrix} = \left(\begin{array}{c|ccc} 0 & 1 & 0 & 0 \\ \hline 1 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 \end{array}\right) \begin{bmatrix} x \\ w_1 \\ w_2 \\ u \end{bmatrix}$$

and let $u = Ky$.

(a) Show that this is a filtering problem and determine $G_{m/w}(s)$ and $G_{y/w}(s)$ such that $H_{z/w} = G_{m/w} - KG_{y/w}$.

(b) Show that *not* all assumptions of Theorem 6.5.1 are satisfied!

(c) (It can be shown that in this case Thm 6.5.1 still solves the problem in that it finds the stable causal K that minimizes $\|H_{z/w}\|_{\mathbb{H}_2}$.) Discard the assumptions of Thm 6.5.1 and compute the $\mathbb{H}_2$-optimal filter K.

5. *Chapter 7:* Consider the standard unity feedback system with given controller and uncertain plant

$$K(s) = \frac{1}{s}, \qquad P(s) = \frac{as+b}{s(s^2+s+1)}.$$

The parameters a, b are uncertain. Determine all possible $a, b \in \mathbb{R}$ for which the given controller stabilizes the plant.

6. Consider the $\mathbb{H}_\infty$ filtering problem with

$$G_{m/w}(s) = \frac{1}{s+1}, \qquad G_{y/w}(s) = \frac{s-2}{s+\alpha}$$

in which α is some positive number. Solve the $\mathbb{H}_\infty$ filtering problem. Provide the optimal $K(s)$, the optimal $H_{z/w}(s)$ and optimal norm $\|H_{z/w}\|_{\mathbb{H}_\infty}$.

7. Table 9.1 (page 99) claims that the interconnection matrix for perturbed plant

$$P = (I + V\Delta_P W)P_0$$

is

$$H = -WT_0V.$$

Verify this. Your derivation must be valid for MIMO systems. (For completeness: T_0 is defined as the complementary sensitivity matrix for the nominal plant: $T_0 = (I + P_0K)^{-1}P_0K = P_0K(I + P_0K)^{-1}$.)

problem:	1	2	3	4	5	6	7
points:	4	3+4	4	4+1+4	4	4	4

Grade: $= 1 + 9\frac{p}{p_{\max}}$ (possibly with homework correction of ≤ 0.6)