Robust Control — EXAM

Course code:	191560671
Date:	16-04-2019 (Sports Centre, Hall 2)
Time:	13:45–16:45 (till 17:30 for students with special rights)
Course coordinator & instructor:	G. Meinsma
Type of test:	open book, written exam
Allowed aids during the test:	printed lecture notes, basic calculator

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1. Consider the system $y = Gu$ described by

$$\dot{x} = -2x + u,$$
$$y = x.$$

   (a) Determine $\|G\|_{\mathbb{H}_2}$

   (b) Determine $\|G\|_{\mathbb{H}_\infty}$

2. Consider the standard unity feedback system and assume that $P(s)$ and $K(s)$ are functions (not matrices), and that $P(s)$ is not the zero function. Define Q as

$$Q = \frac{P}{1+PK}.$$

   (a) Show that $KS = \frac{1}{P}(1 - \frac{1}{P}Q)$.

   (b) Suppose that $\frac{1}{P} \in \mathbb{H}_\infty$. Show that the closed loop is internally stable if-and-only-if $Q \in \mathbb{H}_\infty$. [Hint: Theorem 3.4.3 does something similar for *stable* plants.]

3. Suppose the plant is

$$P(s) = \frac{s-1}{s-0.99}.$$

   This is a nasty plant because there is "almost" an unstable pole/zero cancellation in $P(s)$. Assuming we managed to find a stabilizing controller that keeps the magnitude sensitivity function $|S(i\omega)|$ below 0.1 for all $\omega \in [0, 1]$, what can you say about $\|S\|_{\mathbb{H}_\infty}$ and $\|T\|_{\mathbb{H}_\infty}$? (Be as explicit as possible.)

4. The nominal plant $P_0(s) = 1/s^2$ can successfully be controlled with a lead controller of the form $K_0(s) = (2s+1)/(s+2)$. Now suppose the actual plant is $P(s) = (\theta s+1)/s^2$, with θ some uncertain parameter $\theta \in [-0.5, 0.5]$. Does $K_0(s)$ stabilize all possible plants?

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5. Consider the system

$$\dot{x} = ax + u, \qquad x(0) = x_0$$
$$y = x.$$

Here all signals have one entry, and a is an arbitrary real number.

(a) Let $\rho > 0$. Determine the input u that minimizes

$$\int_0^{\infty} y^2(t) + \rho u^2(t)\,\mathrm{d}t$$

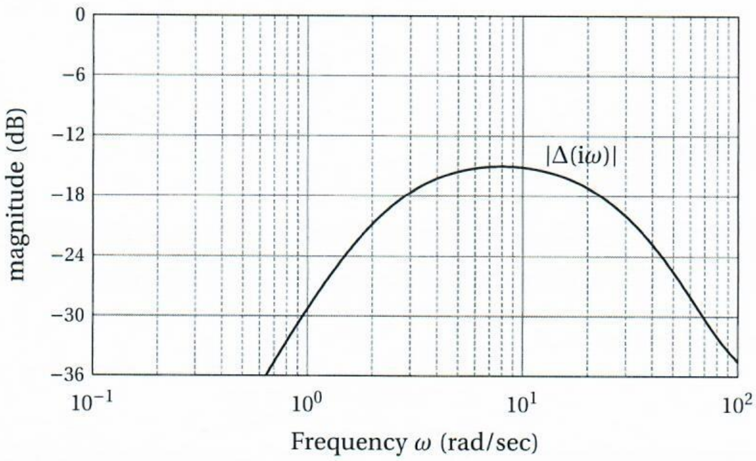
over all inputs u that steer the state to zero as $t \to \infty$.

(b) The u that you constructed in the previous part depends on a, ρ. Does this dependence confirm the generally accepted view that "*the more unstable the plant, the faster we need to control it*"?

6. We consider the *isolated beam* where the temperature on one end is the input, and the temperature on the other end is the output. The relation between input and output is described by a partial differential equation, leading to some stable but *non-rational* transfer function $P(s) = \dfrac{1}{\cosh(\sqrt{s})}$. As nominal *rational* model we choose

$$P_0(s) = \frac{1}{1 + s/2}.$$

Then $\Delta(s) := P(s) - P_0(s)$ is stable and has this magnitude bode plot:



(a) Let $K(s) = 3/s$. Show that it stabilizes the nominal plant $P_0(s)$.

(b) Given $K(s) = 3/s$ determine the interconnection matrix $H_{q/p}(s)$ for this problem.

(c) It can be shown that $\|H_{q/p}\|_{\mathbb{H}_\infty}$ equals 1.965. Are we guaranteed that the controller $K(s) = 3/s$ also stabilizes the isolated beam with transfer function $P(s)$.

problem:	1	2	3	4	5	6
points:	4+4	2+5	4	4	4+2	1+3+3

$p_{max} = 36$

Grade: $g = 1 + 9\frac{p}{p_{\max}}$. The final grade is $\frac{2}{3}g + \frac{1}{3}a$ where a is the grade of the practical assignment.