Robust Control — EXAM (resit)

Course code:	191560671
Date:	02-07-2019 (HB 2E)
Time:	08:45–11:45 (till 12:30 for students with special rights)
Course coordinator & instructor:	G. Meinsma
Type of test:	open book, written exam
Allowed aids during the test:	printed lecture notes, basic calculator

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1. Let $\tau > 0$ and consider the system with transfer function

$$G(s) = \frac{e^{-\tau s}}{s+4}.$$

   (a) Determine $\|G\|_{\mathbb{H}_2}$ for $\tau = 0$

   (b) Show that both $\|G\|_{\mathbb{H}_2}$ and $\|G\|_{\mathbb{H}_\infty}$ do not depend on τ (provided $\tau \geq 0$).

2. Use the machinery of Lemma 3.4.1 to construct a *strictly proper* controller $K(s)$ for plant $P(s) = 1/(s+0.01)^2$ such that $H_{y/r}(s)$ is strictly proper and having a bandwidth 0.1 and $H_{y/r}(0) = 1$.

3. Let g be a positive number, and suppose the loop gain is

$$L(s) = \frac{g}{s}.$$

   Determine the delay margin of this $L(s)$.

4. Suppose the plant is

$$P(s) = \frac{s-11}{s-9},$$

   and suppose we managed to find a stabilizing controller with the property that

$$\max_{\omega \in [0,\omega_1]} |S(i\omega)| < 0.1$$

   for some frequency $\omega_1 > 0$. Show that $\|S\|_\infty \geq 10$.

5. Consider Kalman filtering problem

$$\dot{x} = ax + w_1,$$
$$y = x + \sqrt{\sigma}\, w_2$$

Here all signals have one entry, and a is an arbitrary real number, and σ is a positive tuning parameter.

(a) Determine the Kalman filter for this problem.

(b) The Kalman filter that you constructed in the previous part depends on a, σ. Does this dependence confirm the idea that "*the more unstable the plant, the faster the observer*"?

6. Consider the mixed sensitivity problem in the $\mathbb{H}_\infty$ norm for the plant

$$P(s) = \frac{1}{(s+1)(s+2)}.$$

Suppose we are after a stabilizing controller that achieves a closed loop bandwidth of 5, and, of course, $T(0) = 1$. Explain how you would choose your (initial but sensible) weighting functions $V(s), W_1(s), W_2(s)$.

7. Suppose we have an uncertain plant

$$P(s) = P_0(s)(1 + \frac{1}{2}\Delta(s))$$

in which

$$P_0(s) = \frac{1}{s+1}$$

and Δ is some stable system all of which we know is that $\|\Delta\|_\infty < 1$. Consider the controller

$$K(s) = \frac{1}{s}.$$

(a) Show that this $K(s)$ stabilizes the nominal plant $P_0(s)$.

(b) Determine the interconnection matrix $H_{q/p}(s)$ for this problem.

(c) Are we guaranteed that the controller stabilizes all possible plants?

problem:	1	2	3	4	5	6	7
points:	4+4	4	4	4	4+2	4	1+3+2

Grade: $g = 1 + 9\frac{p}{p_{\max}}$. The final grade is $\frac{2}{3}g + \frac{1}{3}a$ where a is the grade of the practical assignment.