Robust Control — EXAM

Course code:	191560671
Date:	20-04-2023 (RA-4334)
Time:	08:45–11:45 (till 12:30 for students with special rights)
Course coordinator & instructor:	G. Meinsma
Type of test:	open book, written exam
Allowed aids during the test:	lecture notes ‘Robust Control” & basic calculator

i. Which Msc programme (AM, BME, EE, SC, ?) do you follow?

ii. In what year did you complete the practical assignment, and with whom (if any) did you do the assignment?

1. Consider the transfer function

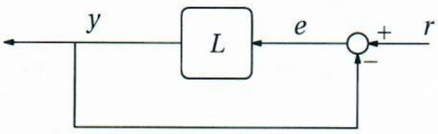
$$G(s) = \frac{-e^{-s}}{s+4}$$

   (a) Determine $\|G\|_{\mathbb{L}_\infty}$.

   (b) Determine $\|G\|_{\mathbb{H}_\infty}$.

   (c) Determine $\|G\|_{\mathbb{H}_2}$.

2. Consider the system



   with loop gain

$$L(s) = \frac{k e^{-s\tau}}{s}.$$

   The loop gain depends on a gain $k \in \mathbb{R}$ and a nonnegative delay τ.

   (a) Suppose first that $\tau = 0$. Determine all $k \in \mathbb{R}$ for which the closed loop is internally stable

   (b) Let $\tau > 0$. Determine all k for which the closed loop is internally stable.

   (c) Let $\tau = 2$ and suppose k is such that the closed loop is internally stable. Sketch a reasonable response y to a step input $r(t) := \mathbb{1}(t)$.

3. Suppose the plant is

$$P(s) = \frac{s-2}{s^2-9}.$$

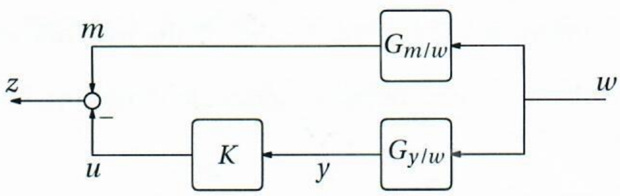
   Assuming we managed to find a stabilizing controller that keeps the magnitude sensitivity function $|S(i\omega)|$ below 0.1 for all $\omega \in [0,2]$, what can you say about $\|S\|_{\mathbb{H}_\infty}$ and $\|T\|_{\mathbb{H}_\infty}$? (Be as explicit as possible.)

4. Consider the $\mathbb{H}_2$ filtering problem with state representation

$$\begin{bmatrix} \dot{x} \\ z \\ y \end{bmatrix} = \left[\begin{array}{c|cc} -2 & 1 & 0 \\ \hline 1 & 0 & -1 \\ -3 & 1 & 0 \end{array}\right] \begin{bmatrix} x \\ w \\ u \end{bmatrix}.$$

Determine the stable causal filter K that minimizes $\|H_{z/w}\|_{\mathbb{H}_2}$ over all stable causal filters.

5. Consider the $\mathbb{H}_\infty$ filtering problem



with

$$G_{m/w}(s) = \frac{1}{s+2}, \qquad G_{y/w}(s) = \frac{s-1}{s+2}$$

Determine the stable causal filter K that minimizes $\|H_{z/w}\|_{\mathbb{H}_\infty}$ over all stable causal filters.

6. Let W_S^{-1}, W_P be two stable rational transfer functions. Suppose our plant P is uncertain: $P = P_0 + \Delta$ where Δ is some unknown stable system that is bounded by

$$|\Delta(\mathrm{i}\omega)| \leq |W_P(\mathrm{i}\omega)| \qquad \forall \omega \in \mathbb{R}.$$

Suppose we want to design a controller that stabilizes the closed loop and achieves the performance specification that

$$|S(\mathrm{i}\omega)| \leq |W_S(\mathrm{i}\omega)| \qquad \forall \omega \in \mathbb{R}$$

for all possible plants $P = P_0 + \Delta$. The lecture note claims that all this is guaranteed if

$$\|H\|_{\mathbb{H}_\infty} < 1$$

for some appropriate interconnection matrix H depending on P_0 and W_P, W_S. Determine this H.

problem:	1	2	3	4	5	6
points:	3+2+4	2+4+2	5	6	4	4

Grade: $g = 1 + 9\frac{p}{p_{\max}}$. The final grade is $\frac{3}{4}g + \frac{1}{4}a$ where a is the grade of the practical assignment.

Robust Control — EXAM

Course code:	191560671
Date:	04-07-2023 (Therm 1)
Time:	08:45–11:45 (till 12:30 for students with special rights)
Course coordinator & instructor:	G. Meinsma
Type of test:	open book, written exam
Allowed aids during the test:	lecture notes 'Robust Control" & basic calculator

i. Which MSc programme (AM, BME, EE, SC, ?) do you follow?

ii. In what year did you complete the practical assignment, and with whom (if any) did you do the assignment?

1. Determine the spectral norm of

$$A = \begin{bmatrix} 1 & 2 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}.$$

2. Determine $\|G\|_{\mathbb{H}_\infty}$ of the system G with state representation

$$\begin{aligned} \dot{x} &= -2x + u \\ y &= x \end{aligned}$$

3. Use the Guillemin-Truxal design procedure to design a proper stabilizing controller K for $P(s) = 1/(s^2 + s + 1)$ such that $H_{y/r}$ has bandwidth $1/2$.

4. Suppose the loop gain $L(s)$ is stable, and that $\operatorname{Re}(L(i\omega)) \geq 0$ for all frequencies.

   (a) Can the closed loop be unstable?

   (b) Assuming the closed loop is stable, what can you say about the phase margin?

5. Let $L(s) = 1/(s - 1/4)$. Determine $\int_0^\infty \ln|S(i\omega)|\, d\omega$.

6. Consider the system

$$\dot{x} = -2x + u, \qquad x(0) = x_0,$$
$$z = -3x + u.$$

Solve the LQ problem with stability, that is, determine the optimal state feedback $u = -Fx$ and minimal cost $\int_0^\infty z^2(t)\,\mathrm{d}t$.

7. The nominal plant $P_0(s) = 1/s^2$ can successfully be controlled with a lead controller of the form $K_0(s) = (2s+1)/(s+2)$. Now suppose the actual plant is $P(s) = (\theta s+1)/s^2$, with θ some uncertain parameter $\theta \in [-0.5, 0.5]$. Does $K_0(s)$ stabilize all possible plants?

8. Table 9.1 from the lecture notes claims that the interconnection matrix for uncertain plant $P = P_0(I + V\Delta_P W)$ is $H = WKS_0P_0V$. Derive this result (and your derivation must also be valued for MIMO systems).

problem:	1	2	3	4	5	6	7	8
points:	3	4	5	3+3	3	5	5	5

Grade: $g = 1 + 9\frac{p}{p_{\max}}$. The final grade is $\frac{3}{4}g + \frac{1}{4}a$ where a is the grade of the practical assignment.