Course	:	Systems & Control (MasterMath)
Date	:	December 6th 2021
Time	:	10:00-13:00
Location	:	Science Park Utrecht

Please provide motivation for all your answers and calculations. This is NOT an open book exam, but you are allowed to use a handwritten 'cheatpaper' of one page A4, single sided. The use of electronic devices is not allowed nor needed.

1. Condider

$$R(\xi) = \begin{bmatrix} \xi & \xi - 1 \\ \xi - 1 & \xi - 3 \end{bmatrix} \qquad \mathfrak{B} = \{w \mid R(\frac{d}{dt})w = 0\}$$

   (a) (3p) Eliminate w_2 to obtain an equation for w_1.

   (b) (3p) Eliminate w_1 to obtain an equation for w_2.

   (c) (3p) Determine the joint behavior of w_1 and w_2, that is, the behavior of w.

   (d) (4p) Now consder $R(\xi) \in \mathbb{R}^{2\times 2}[\xi]$ and let $p(\xi) = \det R(\xi)$. Under what conditions on the entries of $R(\xi)$ can we conclude that the behavior of w_1 after elimination of w_2 and the behavior of w_2 after elimination of w_1 is given by $p(\frac{d}{dt})w_1 = 0$ and $p(\frac{d}{dt})w_2 = 0$ respectively?

2. Consider the system of nonlinear differential equations:

$$\begin{aligned} \frac{d}{dt}x_1 &= -\sin x_1 + x_2 \cos x_2 \\ \frac{d}{dt}x_2 &= -2x_2 e^{-x_2} \end{aligned}$$

   (a) (1p) Check that $\bar{x} = (0, 0)$ is an equilibrium point.

   (b) (3p) Determine the matrix $A \in \mathbb{R}^{2\times 2}$ so that the linearisation about $\bar{x}$ is given by

$$\frac{d}{dt}x = Ax.$$

   (c) (2p) Show that $A + A^{\mathrm{T}}$ is negative definite.

   (d) (3p) Determine a quadratic Lyapunov function and show that $\bar{x}$ is (locally) asymptotically stable.

   (e) (2p) Is $\bar{x}$ the only equilibrium point? Argue that $\bar{x}$ cannot be *globally* asymptotically stable.

3. Consider

$$R(\xi) = \begin{bmatrix} \xi^2+1 & \xi-1 & \xi^2+2\xi \\ \xi^2-1 & \xi+1 & \xi+2 \\ 2\xi^2 & 2\xi & \xi^2+3\xi+2 \end{bmatrix}$$

(a) (3p) Is $R(\xi)$ of full row rank?

(b) (3p) If $R(\xi)$ is of full row rank then calculate its determinant and the behavior represented by $R(\frac{d}{dt})w = 0$.

If $R(\xi)$ is not of full row rank then determine a full row rank representation and subsequently a proper input/output representation. Clearly indicate which part of w serves as output and which as output.

(c) (3p) Is the behavior represented by $R(\frac{d}{dt})w = 0$ controllable?

4. (8p) Consider the i/s/o system given by

$$\begin{aligned} \frac{d}{dt}x &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x. \end{aligned}$$

Determine a dynamic compensator such that the controller poles are -1 and -2 and the observer poles are -2 and -3.

Grade: $1 + \dfrac{9P}{41}$